ANSWERS

1.01

1 D

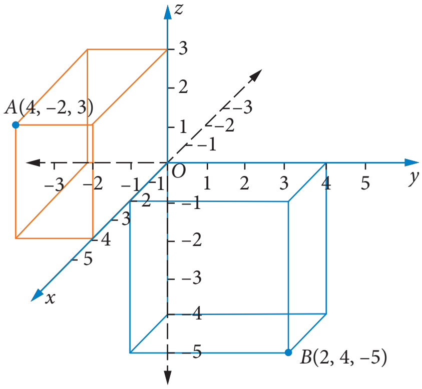
2 B

3 C

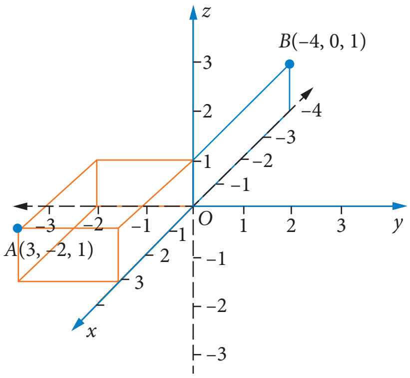
4 A

5 $A(2, 3, -1)$, $B(-2, 5, 4)$, $C(3, -3, 0)$, $D(-4, -4, 3)$, $E(2, 0, 2)$

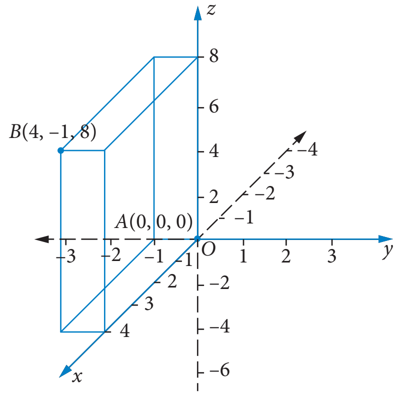
6 a



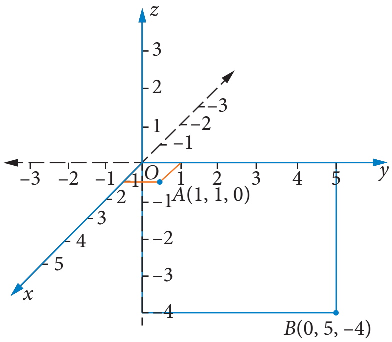
b



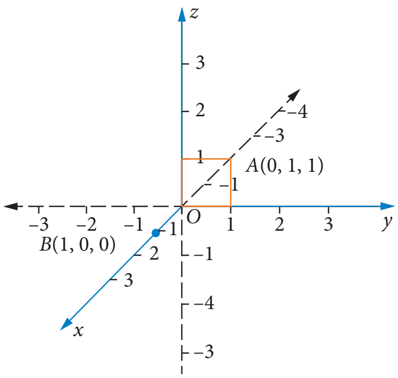
c



d



e



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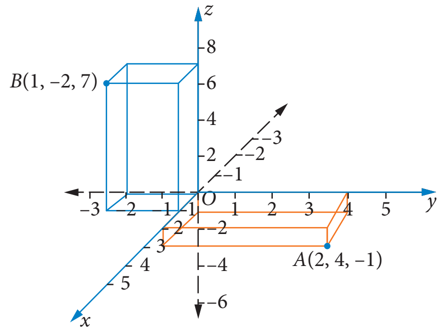
f 

7 a (2, 0, 0), (2, 4, 0), (0, 4, 0), (0, 4, 3), (0, 0, 3), (2, 0, 3)
b (2, 2, 1), (2, 4, 1), (−5, 4, 1), (−5, 2, −3), (−5, 4, −3), (2, 2, −3)

8 a $2\sqrt{26}$ b $\sqrt{53}$ c 9
d $\sqrt{33}$ e $\sqrt{3}$ f $\sqrt{101}$

9 a (3, 1, −1) b $(-\frac{1}{2}, -1, 1)$ c $(2, -\frac{1}{2}, -4)$
d $(\frac{1}{2}, 3, -2)$ e $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ f $(1\frac{1}{2}, 1, 3)$

10 a $\sqrt{38}$ b $3\sqrt{2}$ c $3\sqrt{5}$
d $\sqrt{105}$ e $3\sqrt{5}$ f $\sqrt{101}$

11 a $(2\frac{1}{2}, 0, -\frac{1}{2})$ b $(2, 1\frac{1}{2}, \frac{1}{2})$ c $(0, 2\frac{1}{2}, 3)$
d $(\frac{1}{2}, 1, 2)$ e $(0, 4, 1\frac{1}{2})$ f $(-1, -\frac{1}{2}, 1)$

12 a $x^2 + y^2 + z^2 - 16 = 0$
b $x^2 + y^2 + z^2 - 6y - 141 = 0$
c $x^2 + y^2 + z^2 + 4x - 5 = 0$
d $x^2 + y^2 + z^2 - 6x + 2y - 4z + 5 = 0$
e $4x^2 + 4y^2 + 4z^2 + 40x - 8z + 103 = 0$
f $x^2 + y^2 + z^2 - 4x + 6y - 8z - 20 = 0$

13 a $C = (0, 0, 0), r = 6$ b $C = (5, -3, 3), r = 3$
c $C = (-1, \frac{1}{2}, 0), r = \sqrt{2}$ d $C = (-2, 0, 0), r = 2$
e $C = (4, 0, -4), r = 4$ f $C = (-2, 1, -1), r = 2$

14 a $PR^2 = PQ^2 + QR^2$; Area of $\triangle PQR = \frac{3\sqrt{2}}{2}$ units^2
b $QR^2 = PQ^2 + PR^2$; Area of $\triangle PQR = \frac{9\sqrt{2}}{2}$ units^2

15 $C = (2, 0 -3)$; $r = 2\sqrt{6}$; $x^2 + y^2 + z^2 - 4x + 6z - 11 = 0$

16 $6x - 12y + 4z + 13 = 0$

17 a A plane parallel to the *xy*-plane.
b A plane parallel to the *xz*-plane.
c The *xz*-plane.

1.02

1 C

2 a (5, −1, 12) b (−13, 7, 14) c (−3, 1, 1)
d (1, −1, −2) e (25, −11, −12) f (−16, 9, 28)

3 a $\sqrt{170}$ b $3\sqrt{46}$ c $\sqrt{11}$
d $\sqrt{6}$ e $\sqrt{890}$ f $\sqrt{1121}$

4 **a**, **c**, **d** and **e** are parallel pairs, **b** and **f** are not parallel

5 a **b** − **a** b **c** − **a** c **f** − **d**
d **c** − **f** e **b** − **e**

6 a (−7, −1, −6), (9.3, 188.1°, −40.3°)
b (−8, −3, 8), (11.7, 200.6°, 43.1°)
c (−2, −5, −5), (7.3, 248.2°, −42.9°)
d (3, 0, −5), (5.8, 0°, −59.0°)
e (9, 6, 11), (15.4, 33.7°, 45.5°)

7 a 11.75 b 8.602 c 10.49

8 a (0.4444, −0.7778, 0.4444)
b (−0.8571, 0.2857, −0.4286)
c (−0.7071, −0.4243, 0.5657)

9 C

10 a 2**i** − 3**j** + 4**k**, 5.39 b −3**i** − 2**j** + 4**k**, 5.39
c 3**i** + **j** − 5**k**, 5.92 d −**i** + 4**j** − 4**k**, 5.74
e −3**i** − **j** − 2**k**, 3.74 f −6**i** + 2**j** + 5**k**, 8.06
g 4**i** − 7**j** + 2**k**, 8.31 h **i** + 6**j** + 4**k**, 7.28
i −5**i** − 3**j** + 2**k**, 6.16 j **i** − **k**, 1.41
k 3.624**i** − 8.140**j** − 4.540**k**, 10
l −2.399**i** − 2.399**j** + 2.120**k**, 4
m 0.486**i** + 1.042**j** + 10.940**k**, 11
n 6.928**i** − 12**j** − 8**k**, 16
o −1.327**i** + 2.393**j** − 7.518**k**, 8
p −15.499**i** + 42.583**j** + 21.131**k**, 50
q −28.284**i** − 28.284**j** − 69.282**k**, 80

11 a (7.071, 300.96°, −34.45°)
b (6, 206.6°, 41.81°)
c (8.602, 66.80°, 27.72°)
d (9.644, 255.96°, −31.23°)

12 a (2.326, 1.135, 9.659)
b (−2.779, 4.813, −5.755)
c (8.660, −5, 17.32)
d (−15.59, −8.999, −21.45)
e (−19.60, 19.60, 16)

13 A

14 a (9.2, 115.7°, 1.3°) b (25.6, 350.9°, −55.3°)
c (21.8, 134.1°, 29.7°) d (37.0, 2.0°, −58.0°)
e (22.2, 98.1°, −26.7°)

15 a (26.6, 275.8°, 30.5°) b (48.4, 344.6°, −52.7°)
c (105.7, 187.5°, 58.8°) d (71.0, 332.1°, −45.9°)
e (76.6, 179.6°, 57.4°)

16 **d** = **a** + **c** − **b**, **e** = **f** + **a** − **b**, **g** = **f** + **c** − **b**,
h = **f** + **a** − 2**b** + **c**, **m** = $\frac{1}{2}$(**a** + **c** − **b** + **f**)

17 a 5.764**i** + 5.764**k** + 1.733**j** (km) or 5764**i** + 5764**k** + 1733**k** (m) with **i** as east, **j** as north and **k** as up.
b −1.414**i** + 1.414**j** − 1.5**k** (km) or −1414**i** + 1414**j** −1500**k** (m)
c 4.350**i** + 7.178**j** − 0.233**k** (km) or 4349**i** + 7178**j** − 233**k** (m), or (8.4, 58.8°, 1.6°)

18 A length of 470.2 m allows slack of 23.4 m in the centre of the distance of 467.9 m.

19 Bearing 153.4°, angle of descent 30.8° and distance 781.0 m

20 a 40° to the yacht and 15° upwards.
b 20 000 N
c About 4799 N rearwards , 4903 perpendicular to the yacht and 5176 N upwards.
d About 4799 N.

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e About 5176 N. By 'pointing the sail higher', less upward deflection of the wind is created, less force is directed downwards on the yacht and more wind energy is converted to forward (and sideways) motion of the yacht.

1.03

1 C
2 C
3 B
4 E
5 D
6 C
7 D
8 a 35.46 b 21.21 c 0 d 180
e 12.12 f −57.16 g −70.15 h 48
9 a 44.50 b 46.36 c 57
d 12.99 e 25.90 f 0
10 a −14 b 0 c −96
d 74 e −1 f 0
g −42 h −2 i 2
11 a −30 b −85 c 73
d 17 e 0 f 41
12 a −8 b −1 c −18
d −18 e 9 f 9
13 a 167.9° b 134.7° c 104.3°
14 a $-\frac{\sqrt{30}}{10}$ b $-\frac{15\sqrt{14}}{14}$ c $\frac{\sqrt{10}}{10}$
d $\sqrt{30}$ e **0** f $-\frac{3\sqrt{30}}{5}$
15 a 97.5° b 88.6° c 62.5° d 79.6°
16 a 74 b **8** c 48.2°
d 77° e −7.31 f 5.89
17 a 1 Nm b 60 Nm c 14 Nm
18 72 Nm
19 −14.43 Nm
20 40 Nm
21 $k = 2$
22 a (1, −8, 2)
b $\cos(\alpha) \approx 0.1204$, $\cos(\beta) \approx -0.9631$, $\cos(\gamma) \approx 0.2408$
c $\alpha \approx 83.1°$, $\beta \approx 164.4°$, $\gamma \approx 76.1°$
23 Proof
24 $\cos(\alpha - \beta) = [\cos(\alpha), \sin(\alpha)] \cdot [\cos(\beta), \sin(\beta)]$
$= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
25 a 76.53 N at 13° to the 50 N force
b i 488.9 J ii 231.8 J iii 257.1 J
c 488.9 = 231.8 + 257.1, but because of the angles, the contributions of the 30 N and 50 N forces are almost the same.
26 $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}||\mathbf{a}| \cos(0°)$
$= |\mathbf{a}||\mathbf{a}| \times 1$
$= |\mathbf{a}|^2.$
27 It is less than 30° because, as the altitude goes from 0° to 90°, the angle goes from 30° to 0°.
Changing both vectors to components and finding the angle using the dot product gives an angle of 25.64°.

INVESTIGATION: PROPERTIES OF THE VECTOR PRODUCT

1 $\cos(\theta) = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$

$\sin(\theta) = \sqrt{\frac{(a_1b_2 - a_2b_1)^2 + (a_1b_3 - a_3b_1)^2 + (a_2b_3 - a_3b_2)^2}{(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}}$

$|\mathbf{v}_1 \times \mathbf{v}_2| = \sqrt{(a_1b_2 - a_2b_1)^2 + (a_1b_3 - a_3b_1)^2 + (a_2b_3 - a_3b_2)^2}$

2 They are the same.
3 $[(a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}] \cdot [a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}] = 0$ and $[(a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}] \cdot [b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}] = 0$
4 It is easy to get lost in the detail.

1.04

1 B
2 A
3 D
4 All answers are perpendicular to both vectors.
a 20.78 b 19.56 c −49.25 d 0
e 35.10 f −35 g 77 h 24.25
5 a $-100.2\mathbf{k} = (100.2, 0°, -90°)$
b $-30.04\mathbf{k}$ c $-647.0\mathbf{k}$
d (4.596, 11°, 0°) e (1.654, 240°, 0°)
f (34.47, 291°, 0°)
6 a $-57\mathbf{k}$ b $-126\mathbf{k}$ c $56\mathbf{k}$
d (49, −54, 47) e (19, 31, −52)
f (−86, −159, −90) g (30, 12, −78)
h (83, 17, −6) i (6, −61, −42)
7 a $\begin{bmatrix}0\\0\\-16\end{bmatrix}$ b $\begin{bmatrix}0\\0\\0\end{bmatrix}$ c $\begin{bmatrix}0\\0\\0\end{bmatrix}$
d $\begin{bmatrix}13\\39\\26\end{bmatrix}$ e $\begin{bmatrix}74\\3\\-35\end{bmatrix}$ f $\begin{bmatrix}-63\\-47\\9\end{bmatrix}$
8 a $18\mathbf{i} + 19\mathbf{j} + 11\mathbf{k}$ b $-32\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$
c $21\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$ d $21\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$
e $-13\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$ f $13\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}$
g $38\mathbf{i} - 25\mathbf{j} - 107\mathbf{k}$ h $-21\mathbf{i} + 83\mathbf{j} - 109\mathbf{k}$
9 a $50\mathbf{i} - 17\mathbf{j} - 38\mathbf{k}$ b $-102\mathbf{i} - 17\mathbf{j}$
c $102\mathbf{i} + 17\mathbf{j}$ d $63\mathbf{i} + 126\mathbf{j} - 154\mathbf{k}$
e $-770\mathbf{i} + 1771\mathbf{j} + 1134\mathbf{k}$ f $-168\mathbf{i} - 85\mathbf{j} + 76\mathbf{k}$
g $532\mathbf{i} + 1064\mathbf{j} + 2366\mathbf{k}$ h $64\mathbf{i} + 204\mathbf{j} - 432\mathbf{k}$
i $-340\mathbf{i} + 765\mathbf{j} - 1156\mathbf{k}$ j $-340\mathbf{i} + 765\mathbf{j} - 1156\mathbf{k}$
k $344\mathbf{i} - 831\mathbf{j} + 1276\mathbf{k}$
10 B
11 a 11 unit^2 b 22.99 unit^2 c 8.66 unit^2
12 a 6 unit^2 b 45.72 unit^2
13 a (24, 2, −20) b (68, −36, 30)
14 $\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a} = (22\mathbf{i} - 14\mathbf{j} - 6\mathbf{k}) + (11\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}) + (-33\mathbf{i} + 21\mathbf{j} + 9\mathbf{k}) = \mathbf{0}$
15 Either $\mathbf{a} = \mathbf{0}$, $\mathbf{b} = \mathbf{0}$ or $\mathbf{b} = k\mathbf{a}$ for some real number k ($\mathbf{a}$ and $\mathbf{b}$ are parallel).
16 a $D(4, 7, 1)$ b $\sqrt{491}$

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17 $0.599\mathbf{i} + 0.799\mathbf{j} + 0.0499\mathbf{k}$

18 e.g. $(-\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) \times (-2\mathbf{i} + 12\mathbf{j} + 9\mathbf{k}) = 102\mathbf{i} + 17\mathbf{j}$ and $(-2\mathbf{i} + 12\mathbf{j} + 9\mathbf{k}) \times (-\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) = -102\mathbf{i} - 17\mathbf{j}$

19 e.g. $(7\mathbf{i} - 4\mathbf{j} + 11\mathbf{k}) \times [(-2\mathbf{i} + 12\mathbf{j} + 9\mathbf{k}) \times (14\mathbf{i} - 7\mathbf{j})] = -770\mathbf{i} + 1771\mathbf{j} + 1134\mathbf{k}$ and $[(7\mathbf{i} - 4\mathbf{j} + 11\mathbf{k}) \times (-2\mathbf{i} + 12\mathbf{j} + 9\mathbf{k})] \times (14\mathbf{i} - 7\mathbf{j}) = 532\mathbf{i} + 1064\mathbf{j} + 2366\mathbf{k}$

1.05

1 $\mathbf{q} = \dfrac{\mathbf{c}+\mathbf{d}}{2}$

$\mathbf{p} = \dfrac{\mathbf{a}+\mathbf{d}}{2}$

$\mathbf{s} = \dfrac{\mathbf{b}+\mathbf{c}}{2}$

$\mathbf{r} = \dfrac{\mathbf{a}+\mathbf{b}}{2}$

$\mathbf{PQ} = \dfrac{\mathbf{c}+\mathbf{d}}{2} - \dfrac{\mathbf{a}+\mathbf{d}}{2}$

$\mathbf{PQ} = \dfrac{\mathbf{c}-\mathbf{a}}{2}$

$\mathbf{RS} = \dfrac{\mathbf{b}+\mathbf{c}}{2} - \dfrac{\mathbf{a}+\mathbf{b}}{2}$

$\mathbf{RS} = \dfrac{\mathbf{c}-\mathbf{a}}{2}$

$\mathbf{PQ} = \mathbf{RS}$

$\mathbf{PQ} \parallel \mathbf{RS}$ and $|\mathbf{PQ}| = |\mathbf{RS}|$

2 Proof

3 Proof

4 Proof

5 $\mathbf{p} = \dfrac{1}{k_1 + k_2}(k_2\mathbf{a} + k_1\mathbf{b})$

6 a $\mathbf{m} = \dfrac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$

b Proof

c $\mathbf{m} = \dfrac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$

7 a $\mathbf{d} = \left(\dfrac{|\mathbf{c}-\mathbf{a}|}{|\mathbf{b}-\mathbf{a}|+|\mathbf{c}-\mathbf{a}|}\right)\mathbf{b} + \left(\dfrac{|\mathbf{b}-\mathbf{a}|}{|\mathbf{b}-\mathbf{a}|+|\mathbf{c}-\mathbf{a}|}\right)\mathbf{c}$

b 0

c The angle bisectors divide the side opposite the vertex in the ratio of the other sides.

8 Proof

9 a $\mathbf{h} = \dfrac{1}{4}(\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d})$ b Proof

10 $\mathbf{h} = \dfrac{1}{4}(\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d})$

11 Proof

12 Proof

13 Proof

14 Proof

15 Demonstration

16 Proof

1.06

1 a $\mathbf{a} = -4\mathbf{i} - 4\mathbf{k}$

b $\mathbf{p} = 3\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + t(-4\mathbf{i} - 4\mathbf{k})$

c The domain is restricted to $0 \le t \le 1$

2 $\mathbf{p} = 3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + t(-6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$, for $0 \le t \le 1$,

$\dfrac{x-3}{-6} = \dfrac{y-7}{-5} = \dfrac{z+2}{4}$

3 $\mathbf{p} = 9\mathbf{i} + \mathbf{j} + 2\mathbf{k} + t(\mathbf{i} + \mathbf{j} + \mathbf{k})$

4 $\dfrac{x}{1} = \dfrac{y}{-5} = \dfrac{z}{3}$

5 $\dfrac{x-1}{-1} = \dfrac{y}{1}$, $z = 1$

6 $\dfrac{x-1}{4} = \dfrac{y-2}{5} = \dfrac{z-1}{-1}$

7 $\mathbf{p} = \mathbf{j} - \mathbf{k} + t(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

8 a $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + t(\mathbf{i} + 3\mathbf{j} - \mathbf{k})$

b $\dfrac{x-2}{1} = \dfrac{y-3}{3} = \dfrac{z-5}{-1}$

9 a $\dfrac{x-1}{-2} = \dfrac{y-2}{-4} = \dfrac{z-3}{-6}$

b $\dfrac{x-1}{3} = \dfrac{y-2}{-6} = \dfrac{z-3}{1}$

10 a $\mathbf{p} = \mathbf{i} - \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $\dfrac{x-1}{2} = \dfrac{y}{2} = \dfrac{z+1}{-1}$

b $\mathbf{p} = \mathbf{j} - \mathbf{k} + t(\mathbf{i} + 2\mathbf{j})$, $\dfrac{x}{1} = \dfrac{y-1}{2}$, $z = -1$

c $\mathbf{p} = -6\mathbf{i} + 5\mathbf{j} + 2\mathbf{k} + t(2\mathbf{i} - 5\mathbf{k})$, $\dfrac{x+6}{2} = \dfrac{z-2}{-5}$, $y = 5$

d $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + t(-\mathbf{i} + 3\mathbf{k})$, $\dfrac{x-2}{-1} = \dfrac{z+3}{3}$, $y = 3$

11 a $\mathbf{p} = -4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + t(2\mathbf{i} + \mathbf{j} - 4\mathbf{k})$,

$\dfrac{x+4}{2} = y - 3 = \dfrac{z-5}{-4}$

b $\mathbf{p} = 5\mathbf{j} + t(5\mathbf{i} - 5\mathbf{j} - 5\mathbf{k})$, $x = 5 - y = -z$

c $\mathbf{p} = 9\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + t(-5\mathbf{i} + 2\mathbf{j} + \mathbf{k})$,

$\dfrac{x-9}{-5} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$

d $\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + t(-\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$,

$\dfrac{x-2}{-1} = \dfrac{y+1}{5} = \dfrac{z-3}{-6}$

12 Q lies on the line.

13 a $\mathbf{p} = \mathbf{i} - \mathbf{j} + t(5\mathbf{i} - \mathbf{j} - \mathbf{k})$

b Q lies on the line when $t = -3$

14 a $y = (x+4)^3 - 2$, a cubic

b $x^2 + y^2 = 1$, a circle of radius 1

c $\dfrac{x^2}{4^2} + \dfrac{y^2}{3^2} = 1$, an ellipse with a major axis of length 8 and minor axis of length 6.

15 $(1, 1, -2)$

16 a $\mathbf{p} = \mathbf{i} - 2\mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

b $\mathbf{p} = 8\mathbf{i} + \mathbf{j} + 4\mathbf{k} + t(4\mathbf{i} + \mathbf{j} + \mathbf{k})$

c $\mathbf{p} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} + t(-2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

17 a Lines intersect at (1, 1, 1) (when $t = -2$ and $s = 3$)
b Demonstration
c Skew lines.

18 a $\mathbf{p} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 4\mathbf{k})$,
$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{-4}$
b Q and R lie on the line.

19 (8, −5, 0)

20 They cross at (14, 25, 18) at 9 seconds, so they collide.

21 They would get within about 220 m about 43.4 s later unless they take avoidance action.

22 The first aircraft is circling the airport at a height of 6000 m and a horizontal distance of 10 km from the airport. The second is circling at a height of 8000 m and a horizontal distance of 12 km and the third is landing at the airport in 6 minutes from the direction S 33.7° W.

1.07

1 C
2 E
3 P and R are on the plane.
4 **a** and **c**
5 a $2x + 10y - 7z + 4 = 0$
b $53z^2 + 104y^2 - 140yz + 154y - 144z < 0$
c $104y^2 + 53z^2 - 140yz + 208y - 140z \geq 220$
6 a $2x + y - 3z + 5 = 0$
b $5x^2 + 10z^2 - 12xz + 24x - 40z \leq 24$
c $5x^2 + 10z^2 - 12xz + 26x - 48z + 34 > 0$
7 $x + 2y + 3z - 13 = 0$
8 $2x + 4y - 5z = 0$
9 a $[(x-3)\mathbf{i} + (y-2)\mathbf{j} + (z-5)\mathbf{k}] \cdot (\mathbf{i} + \mathbf{k}) = 0$
b $x + z = 8$
10 $11x - 2y - 6z - 69 = 0$
11 B
12 $3x + y - 6z + 5 = 0$
13 $6x - 5y - z + 84 = 0$
14 a $z = 2$ b $x = 5$ c $y = -3$
15 $3x - y + 2z + 11 = 0$
16 $2x + y + z - 4 = 0$
17 a $2x - 8y + 5z - 18 = 0$
b $9x - y - z - 8 = 0$
c $14x - 7y - 8z - 52 = 0$
d $y - 1 = 0$
e $20x - 5y + 2z = 0$
f $x + 2y - 4z + 7 = 0$
18 $4x + 10y - 7z + 2 = 0$
19 $x + 2y - 2z - 1 = 0$
20 $x - y + 2z = 3$
21 $\frac{5\sqrt{5}}{3}$
22 $\frac{5\sqrt{14}}{14}$
23 The normals are parallel and therefore so are the planes.
24 $\frac{26\sqrt{105}}{105}$
25 $71x - 15y + 17z + 10 = 0$
26 The normals to the planes are parallel and so the planes are parallel.
27 The planes intersect in the line
$\mathbf{p} = (3\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$

1.08

1 a $a = 5, b = -3, c = 2$
b Inconsistent
2 a $x = 2, y = 3, z = -5$
b $u = -4, v = 5, w = 3$
c $e = 2, f = -2, g = -1, h = 3$
d $b = -1, c = -2, d = 2, e = 1$
e $f = 2, g = -1, h = 3, k = -5$
3 Adults \$11, children \$5, seniors \$7
4 Gravel 6 t/m^3, sand 5 t/m^3, cement 0.8 t/m^3, water 1 t/m^3
5 Shares = \$60 000, Unit trust = \$68 000, First mortgage = \$72 000

CHAPTER 1 REVIEW

1 B
2 C
3 B
4 A
5 B
6 C
7 C
8 E
9 A
10 a $(-\frac{3}{2}, -1, \frac{1}{2})$, $\sqrt{114} \approx 10.68$
b $(\frac{1}{2}, \frac{3}{2}, -4)$, $\sqrt{106} \approx 10.3$
c $(3, -\frac{5}{2}, -\frac{3}{2})$, $\sqrt{122} \approx 11.05$
11 $x^2 + y^2 + z^2 - 6x + 8y + 4z + 4 = 0$
12 Radius = 3, centre = (3, −1, 2).
13 a $\mathbf{AB} = \mathbf{i} - 3\mathbf{j} + 9\mathbf{k}$, $|\mathbf{AB}| = \sqrt{91}$
b $\hat{\mathbf{p}} = -\frac{3\sqrt{14}}{14}\mathbf{i} + \frac{\sqrt{14}}{14}\mathbf{j} + \frac{\sqrt{14}}{7}\mathbf{k}$
14 a (−3, 2, −3) b (1, −4, 13)
c (−7, 2, −7) d (9, −18, 13)
e (22, −4, 26)
15 a (7.62, 293.2°), (5.39, 326.3°, 48.0°)
b (−6.93, −4), (−15.20, 5.53, −11.76)
c 13, 25.48
16 a $5\mathbf{i} - 7\mathbf{j}$ b $21.65\mathbf{i} - 12.5\mathbf{j}$
c $7\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ d $13\mathbf{i} + 22.5\mathbf{j} + 15\mathbf{k}$

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17 a $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, $\sqrt{26} \approx 5.099$
b $-3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$, $\sqrt{38} \approx 6.164$
c $\mathbf{i} - 3\mathbf{j}$, $\sqrt{10} \approx 3.162$
d $2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, $\sqrt{38} \approx 6.164$
e $-1.828\mathbf{i} - 3.167\mathbf{j} + 3.41\mathbf{k}$, 5
f $6.333\mathbf{i} + 12.429\mathbf{j} - 32.862\mathbf{k}$, 35.7

18 (38.13, 90.3°, 13.2°)

19 a 10.35 b −2 c −1

20 a 71.28 perpendicular to the plane of the vectors
b $342.35\mathbf{k}$ c (23, −10, 19) d $-6\mathbf{i} - 7\mathbf{j} - 9\mathbf{k}$

21 a 8 b −10 c $8\mathbf{i} + \mathbf{j} - 3\mathbf{k}$
d −2 e −10

22 a −2.62 b 0.49 c 94.1°

23 65 Nm

24 a 78.0 b $-34\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}$

25 a $\begin{bmatrix} -28 \\ -68 \\ 74 \end{bmatrix}$ b 178 c $\begin{bmatrix} 20 \\ 0 \\ 16 \end{bmatrix}$ d $\begin{bmatrix} 27 \\ -16 \\ -4 \end{bmatrix}$

e $\begin{bmatrix} -27 \\ 16 \\ 4 \end{bmatrix}$ f $\begin{bmatrix} -88 \\ -184 \\ 136 \end{bmatrix}$ g $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ h $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

26 7.23 units^2

27 (−14, 16, 14)

28 $\mathbf{p} = 3\mathbf{i} - 7\mathbf{j} + 4\mathbf{k} + t(-8\mathbf{i} + 15\mathbf{j} - 2\mathbf{k})$; $\dfrac{x-3}{-8} = \dfrac{y+7}{15} = \dfrac{z-4}{-2}$

29 a $\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k} + t(\mathbf{i} + 7\mathbf{j} - 9\mathbf{k})$; $\dfrac{x-2}{1} = \dfrac{y+1}{7} = \dfrac{z-5}{-9}$
b $\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k} + t(\mathbf{i} + 7\mathbf{j} - 9\mathbf{k})$ for $0 \le t \le 1$
c No

30 $\mathbf{p} = -1\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$; (−3, −8, 9) satisfies the equation for $t = 2$.

31 (5, 6, −2)

32 a $x + y + z - 1 = 0$ b No
c $2y^2 + 2yz + 2z^2 + 2y - 2z < 23$

33 $5x - 2y + 3z - 8 = 0$.

34 a $9x - y - z - 8 = 0$
b $14x - 7y - 8z - 52 = 0$

35 a $x = -2$, $y = \dfrac{1}{3}$, $z = 4$
b Inconsistent, no solutions, the line of intersection of two planes is parallel to the third.
c Infinite solutions, $p = 9 - 2c$, $q = 3 - c$, $r = c$, corresponding to a straight line intersection

36 83 units^3

37 Proof

38 Proof

39 e.g. $(2, 4, -6) \cdot (7, 4, 5) = 0$ and $(2, 4, -6) \times (1, 2, -3) = \mathbf{0}$

40 Prices are: 40 mm, \$15.40; 50 mm, \$21.60; 100 mm, \$53.40

41 Proof

42 Proof

43 Proof

2.01

1 a $x = -1 \pm i\sqrt{2}$ b $x = 2 \pm i\sqrt{3}$
c $z = 3i, 2i$ d $w = \dfrac{-1}{2} \pm i\dfrac{\sqrt{7}}{2}$

2 a $x = 1 \pm i$ b $y = -2 \pm i$
c $z = 3 \pm 2i$ d $w = -4 \pm i\sqrt{2}$

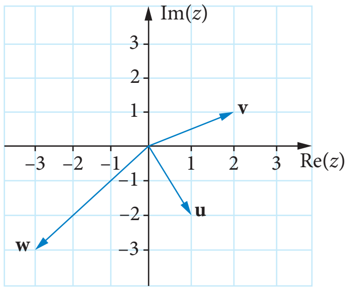
3 a $-i$ b 1 c −1 d 0

4 a $3 + 5i$ b 34

5 a $z^2 - 4z + 5 = 0$ b $z^2 - 2\sqrt{3}\,z + 7 = 0$
c $z^2 + 2z + 6 = 0$

6 $p - qi$

7

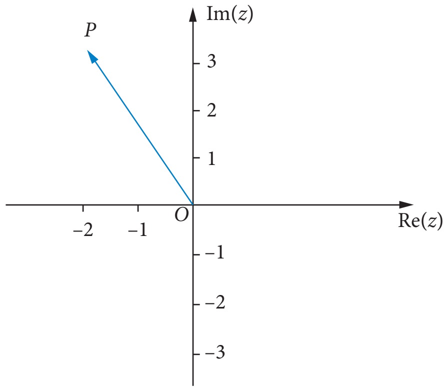


8 $\Delta < 0$ and the coefficients (negative sum and product of the roots) are real, so yes. $x = -1 \pm 2i$

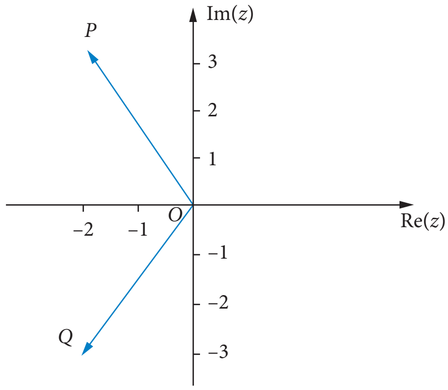
9 Use substitution or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$. The coefficients are not all real.

10 $1 + i$

11 a



b



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c $\overrightarrow{OQ}$ is the reflection of $\overrightarrow{OP}$ in the x-axis.

d $\overrightarrow{OP}-\overrightarrow{OQ}=-2+3i-(-2-3i)=6i$ which is purely imaginary.

2.02

1 a $14+i$ b $13-15i$ c $40+42i$

d 11 e $47+13i$

2 a $u=5, v=-3$ b $u=-16, v=-14$

3 a $-3+14i$ b $6+6i$

c $22-89i$ d $-10+10i$

4 a $\frac{2+i}{5}$ b $\frac{-1+8i}{13}$ c $\frac{2-i\sqrt{5}}{3}$

5 Proof – realise the denominator

6 $=-1$, which is real.

7 a $\text{Re}(z)=-2\sqrt{3}, \text{Im}(z)=-3\sqrt{2}$

b $\text{Re}(z)=13, \text{Im}(z)=-9$

c $\text{Re}(z)=x-4w, \text{Im}(z)=v-y$

d $\text{Re}(z)=\frac{1}{3}, \text{Im}(z)=-\frac{4\sqrt{2}}{3}$

8 $\text{Re}(z)=\frac{(x-1)^2-y^2}{(x-1)^2+y^2}, \text{Im}(z)=\frac{2(x-1)y}{(x-1)^2+y^2}$

9 $\frac{u^2+2uvi-v^2}{u^2+v^2}$

10 $\frac{2(x^2-y^2)}{(x^2+y^2)^2}$

2.03

1 a $z=5[\cos(\pi)+i\sin(\pi)]$

b $z=4\left[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right]$

c $2\left[\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)\right]$

d $v=3\sqrt{2}\left[\cos\left(\frac{-\pi}{7}\right)+i\sin\left(\frac{-\pi}{7}\right)\right]$

2 a $\theta=\frac{\pi}{4}, r=2$ b $\theta=\frac{5\pi}{6}, r=2\sqrt{2}$

c $\theta=\frac{-\pi}{9}, r=1$ d $\theta=\frac{-\pi}{3}, r=\frac{1}{\sqrt{3}}$

3 a $\text{mod}(z)=2, \arg(z)=\frac{\pi}{6}$

b $\text{mod}(z)=3\sqrt{2}, \arg(z)=\frac{\pi}{4}$

c $\text{mod}(z)=\frac{\sqrt{2}}{2}, \arg(z)=\frac{-\pi}{4}$

d $z=-\frac{1}{2}+\frac{i\sqrt{3}}{2}$, $\text{mod}(z)=1, \arg(z)=\frac{2\pi}{3}$

e $z=7i$, $\text{mod}(z)=7, \arg(z)=\frac{\pi}{2}$

f $z=-6$, $\text{mod}(z)=6$, $\arg(z)=\pi$

4 a $z=2\text{ cis}\left(\frac{\pi}{4}\right)$ b $w=2\text{ cis}\left(\frac{2\pi}{3}\right)$

c $u=\text{cis}\left(\frac{-\pi}{6}\right)$ d $v=\text{cis}\left(\frac{-3\pi}{4}\right)$

e $z=\sqrt{5}\text{ cis}\left(\frac{-\pi}{2}\right)$ f $w=\text{cis}(0)$

5 a $6+6i\sqrt{3}$ b $-1+i$

c i d $\frac{-\sqrt{3}}{4}+\frac{i}{4}$

6 a $4\sqrt{3}-4i$ b $\frac{3}{2}-\frac{3i\sqrt{3}}{2}$

c -9 d $\frac{3\sqrt{3}}{2}+\frac{9}{2}i$

7 a $\sqrt{3}+\frac{3\sqrt{3}}{2}i$ b $-2+i\sqrt{2}$

8 a $r[\cos(-\theta)+i\sin(-\theta)]$

b $r[\cos(\pi-\theta)+i\sin(\pi-\theta)]$

c $r[\cos(\theta-\pi)+i\sin(\theta-\pi)]$

9 a Let $x=r\cos(\theta)$, $y=r\sin(\theta)$, then eliminate θ.

b $r\left[\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)\right]$

c Eliminate θ.

2.04

1 a $\text{cis}\left(\frac{12\pi}{35}\right)$ b $\text{cis}\left(\frac{\pi}{3}\right)$

c $\text{cis}(5\alpha)$ d $\text{cis}(-10\beta)$

2 a $\text{cis}\left(\frac{-\pi}{2}\right)$ b $\text{cis}\left(\frac{-3\pi}{4}\right)$

c $3\text{ cis}\left(\frac{-\pi}{5}\right)$ d $2\text{ cis}\left(\frac{\pi}{4}\right)$

3 $\text{cis}\left(\frac{-22\pi}{35}\right)$

4 a $\sqrt{10}\text{ cis}\left(\frac{7\pi}{12}\right)$ b $8\text{ cis}\left(\frac{-6\pi}{7}\right)$

c $-3\text{ cis}\left(\frac{3\pi}{8}\right)=3\text{ cis}\left(\frac{-5\pi}{8}\right)$ d $2\sqrt{3}\text{ cis}\left(\frac{5\pi}{9}\right)$

5 a $\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)$

b $\cos\left(\frac{-3\pi}{7}\right)+i\sin\left(\frac{-3\pi}{7}\right)$

c $\frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right]$

6 a $3\text{ cis}\left(\frac{\pi}{6}\right)$ b $4\text{ cis}(\pi)$ c $3\text{ cis}\left(\frac{2\pi}{5}\right)$

d $\text{cis}(-\theta)$ e $\frac{1}{3}\text{cis}\left(\frac{\pi}{2}\right)$ f $4\text{ cis}\left(\frac{-3\pi}{4}\right)$

7 a $\sqrt{3}\text{ cis}\left(\frac{-11\pi}{12}\right)$ b $\sqrt{3}\text{ cis}\left(\frac{-5\pi}{12}\right)$

c $\frac{1}{\sqrt{3}}\text{cis}\left(\frac{11\pi}{12}\right)$ d $3\text{ cis}\left(\frac{2\pi}{3}\right)$

e $\frac{1}{3}\text{cis}\left(\frac{-\pi}{6}\right)$ f $\sqrt{3}\text{ cis}\left(\frac{11\pi}{12}\right)$

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8 a $\text{cis}\left(\frac{2\pi}{3}\right)$ b $\text{cis}(-2)$ c $12\ \text{cis}\left(\frac{\pi}{4}\right)$

9 Proof

10 Proof

11 Proof

2.05

1 a $\arg(z)=\frac{-\pi}{4}, |z|=\sqrt{2}$ b $\arg(z)=\frac{\pi}{6}, |z|=2$

c $\arg(z)=\frac{3\pi}{4}, |z|=2$ d $\arg(z)=\frac{-2\pi}{3}, |z|=1$

2 a $\arg(z)=-0.381$ b $\arg(z)=2.16$

c $\arg(z)=-2.28$ d $\arg(z)=-1.15$

3 a $\frac{-\pi}{4}$ b 3 c $\theta+\alpha$ d $\sqrt{5}$

4 a $\cos(11\alpha)+i\sin(11\alpha)$

b $4\sqrt{3}\left[\cos\left(\frac{11\pi}{36}\right)+i\sin\left(\frac{11\pi}{36}\right)\right]$

c $3\sqrt{5}\left[\cos\left(\frac{4\pi}{7}\right)+i\sin\left(\frac{4\pi}{7}\right)\right]$

d $\frac{1}{48}\left[\cos\left(\frac{27\pi}{55}\right)+i\sin\left(\frac{27\pi}{55}\right)\right]$

5 a 1 b 81 c $\frac{1}{7\sqrt{7}}$

6 a 6θ b $\frac{-5\pi}{8}$ c $\frac{2\pi}{9}$

7 a $2\sqrt{2}\,\text{cis}\left(\frac{\pi}{12}\right)$ b $\text{cis}\left(\frac{-\pi}{12}\right)$

c $\frac{1}{6}\text{cis}\left(\frac{-3\pi}{4}\right)$ d $\sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$

8 a $2\sqrt{2}\,\text{cis}\left(\frac{7\pi}{12}\right)$ b $\frac{1-\sqrt{3}}{2\sqrt{2}}$

9 $\frac{-17}{\sqrt{290}}$

10 Proof

2.06

1 a $\cos(9\theta)+i\sin(9\theta)$ b $\cos(2\theta)-i\sin(2\theta)$

c $\cos(4\theta)-i\sin(4\theta)$ d $\cos(6\theta)+i\sin(6\theta)$

2 a -16 b $\frac{-81}{2}-\frac{81i\sqrt{3}}{2}$

c $\frac{i}{4}$ d $\frac{625}{2}+\frac{625i\sqrt{3}}{2}$

3 $\cos\left(\frac{2\pi}{5}\right)-i\sin\left(\frac{2\pi}{5}\right)$

4 a $-2+2i$ b $-8+8i\sqrt{3}$

c $64\sqrt{2}+64i\sqrt{2}$ d i

5 a $\frac{i}{512}$ b $-\frac{1}{512}+i\frac{\sqrt{3}}{512}$

6 Proof

7 a $\cos(3\theta)-i\sin(3\theta)$

b $\cos(3\alpha+4\beta)+i\sin(3\alpha+4\beta)$

c $\cos(\phi)-i\sin(\phi)$

d $\cos(17\beta)-i\sin(17\beta)$

8 Proof

9 Proof

10 $2\cos(n\theta)$

INVESTIGATION: EULER AND $e^{i\theta}$

Use index laws for each proof.

2.07

1 a $[\cos(\theta)+i\sin(\theta)]^3=\cos^3(\theta)+3i\cos^2(\theta)\sin(\theta)-3\cos(\theta)\sin^2(\theta)-i\sin^3(\theta)$

b i $\cos(3\theta)=4\cos^3(\theta)-3\cos(\theta)$

ii $\sin(3\theta)=3\sin(\theta)-4\sin^3(\theta)$

2 a $\cos(2\theta)+i\sin(2\theta)$

$=\cos^2(\theta)+2i\cos(\theta)\sin(\theta)-\sin^2(\theta)$

b Proofs

3 Proofs

4 a $\tan(3\theta)=\frac{\sin(3\theta)}{\cos(3\theta)}$ b Proof c Proof

5 Proof

6 $\text{cis}(60°)=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ and $\text{cis}(45°)=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$,

$\cos(15°)=\frac{\sqrt{2}+\sqrt{6}}{4}$

7 a Proof b $\frac{8-5\sqrt{2}}{3}$

8 $-\frac{7\sqrt{3}}{32}+\frac{\pi}{8}$

9 Proof

10 a $2i\sin(\theta)$ b $2i\sin(2\theta)$ c $2i\sin(n\theta)$

CHAPTER 2 REVIEW

1 C

2 B

3 D

4 E

5 A

6 $32-10i$

7 $\frac{9+7i}{5}$

8 $\frac{1+4i\sqrt{3}}{49}$ a $\frac{1}{49}$ b $\frac{4\sqrt{3}}{49}$

9 $2\sqrt{5}\left[\cos\left(\frac{8\pi}{17}\right)+i\sin\left(\frac{8\pi}{17}\right)\right]$

10 $2\left[\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)\right]$

11 $2+2i\sqrt{3}$

12 a $\frac{-11\pi}{18}$ b 30

13 $\frac{1-i}{\sqrt{2}}$

14 a $\cos(A \pm B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
b $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$

15 $\cos(3\theta) + i\sin(3\theta) = \cos^3(\theta) + 3i\cos^2(\theta)\sin(\theta) - 3\cos(\theta)\sin^2(\theta) - i\sin^3(\theta)$, 0.0483

16 $-2^{15} + 0i$

17 Proof

18 a $\cos(5x) - i\sin(5x)$ b $\frac{3}{4}\text{cis}\left(\frac{3\pi}{10}\right)$
c $3[\cos(3\beta) - i\sin(3\beta)]$

19 $2\left[\cos\left(\frac{-3\pi}{4}\right) + i\sin\left(\frac{-3\pi}{4}\right)\right]$
a $\frac{3\pi}{4}$ b $\frac{1}{2}$

20 a $\frac{1}{16}\left[\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right]$
b $\frac{1}{4\sqrt{2}}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$

21 a $\text{cis}\left(\frac{5\pi}{7}\right)$ b $\text{cis}(91\alpha)$

22 $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$, $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
a Proof
b Hint: let $\alpha = 15°$ and use $\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)}$.

3.01

1 a $g[f(x)] = 2x - 5$ b $g[f(x)] = e^{2x} + 2$
c $g[f(x)] = \log_e(e^{2x}) = 2x$ d $g[f(x)] = \sin(2x)$

2 a $f[g(x)] = 2x - 4$ b $f[g(x)] = e^{x^2+2}$
c $f[g(x)] = e^{2\log_e(x)} = x^2$ d $f[g(x)] = 2\sin(x)$

3 a $g(x) = x + c$, $f(x) = x + 3 - c$ where c is a constant
b $g(x) = x^3$, $f(x) = 2x + 5$
c $g(x) = x^2 - 7$, $f(x) = x + 3$
d $g(x) = e^x$, $f(x) = x^2$
e $g(x) = x^2$, $f(x) = \sin(x)$
f $g(x) = \sin(x)$, $f(x) = 2x$

4 $g \circ f$: $g(x) = 4x$, $f(x) = x^2$; $g(x) = x^2$, $f(x) = 2x$; $g(x) = 4x - 16$, $f(x) = x^2 + 4$

5 a $g \circ f = \sin(2x)$; Domain: $x \in \boldsymbol{R}$, Range: $-1 \le y \le 1$
b $g \circ f = \log_e(-x)$; Domain: $x < 0$, Range: $y \in \boldsymbol{R}$
c $g \circ f = 2\sin\left(\frac{1}{2x}\right)$; Domain: $x \in \boldsymbol{R}$, $x \ne 0$, Range: $-2 \le y \le 2$

6 There are many possibilities, e.g. $f(x) = \sqrt{x}$, $g(x) = x^2$; $f(x) = x + 1$, $g(x) = x - 1$

INVESTIGATION: ONE-TO-ONE FUNCTIONS

The horizontal and vertical lines pass through each of the first three graphs at most once.
The horizontal lines pass through some parts of each of the second three graphs more than once, but the vertical lines pass through at most once.
C and D are 1 : 1 functions; A and B are not functions; E and F are not 1 : 1.

3.02

1 a NOT one-to-one b One-to-one
c One-to-one d NOT one-to-one

2 Proof

3 a Strictly decreasing, and one-to-one
b Neither increasing or decreasing and NOT one-to-one
c Neither increasing nor decreasing, NOT one-to-one
d Strictly decreasing, and one-to-one

4 Proof

5 For example, $g \circ f$ with $f(x) = x^3$, $g(x) = 3x - 2$

6 For example, $g \circ f$ with $f(x) = \pi - x$, $g(x) = x + \cos(x)$

7 For example, $g \circ f$ with $f(x) = e^x$, $g(x) = \log_e(x)$

3.03

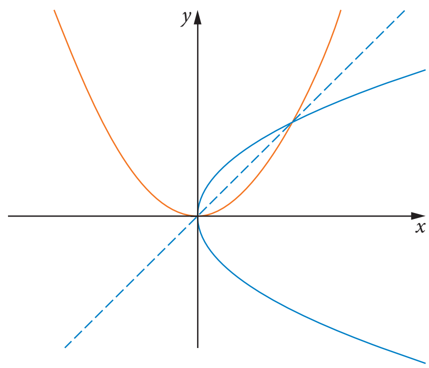
1 a $f^{-1}(x) = \frac{x+1}{2}$ b $f^{-1}(x) = \pm\sqrt{x-1}$
c $f^{-1}(x) = 1 + e^{\frac{x}{2}}$ d $f^{-1}(x) = (x-1)^2$
e $f^{-1}(x) = \sqrt[3]{x-1}$

2 Inverse functions exist for **a**, **c**, **d**, **e**

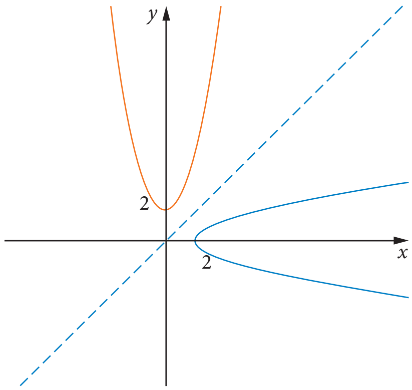
3 Either $x > 0$ or $x < 0$.

4 a $f^{-1}(x) = \sqrt[3]{x}$ b $f^{-1}(x) = \frac{x+2}{3}$
c $f^{-1}(x) = \log_e(x)$ d $f^{-1}(x) = \frac{2}{x}$
e $f^{-1}(x) = 1 - \frac{1}{x}$

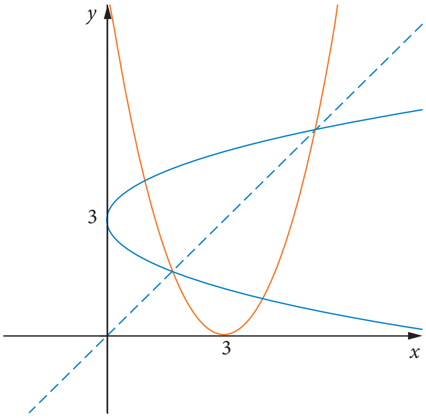
5 a $f^{-1}(x) = \sqrt{x}$, $x \ge 0$, $y \ge 0$
b $f^{-1}(x) = \sqrt{\frac{x+1}{3}}$, $x \ge -1$, $y \ge 0$
c $f^{-1}(x) = \sqrt[4]{x} + 2$, $x \ge 0$, $y \ge 2$
d $f^{-1}(x) = -\sqrt{\frac{3}{x}}$, $x > 0$, $y < 0$
e $f^{-1}(x) = -\sqrt[4]{\frac{2}{x}}$, $x > 0$, $y < 0$

6 Functions in orange, inverse relations in blue.
a


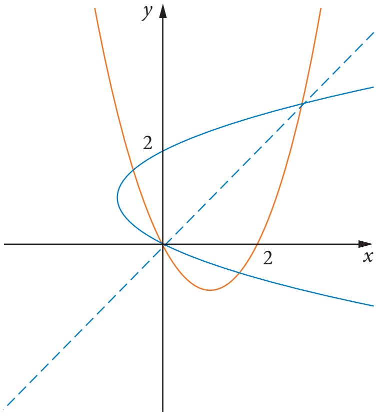
b



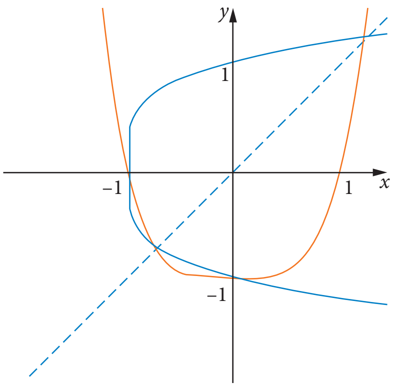
c



d



e



7 a i $f^{-1}(x)=1+\sqrt{x+1}$ ii $f^{-1}(x)=1-\sqrt{x+1}$

b i $f^{-1}(x)=+\sqrt[4]{x+2}$ ii $f^{-1}(x)=-\sqrt[4]{x+2}$

c i $f^{-1}(x)=+\sqrt[4]{\frac{x-1}{2}}$ ii $f^{-1}(x)=-\sqrt[4]{\frac{x-1}{2}}$

d i $f^{-1}(x)=3+\sqrt{x+8}$ ii $f^{-1}(x)=3-\sqrt{x+8}$

e i $f^{-1}(x)=-2+\sqrt{x+7}$ ii $f^{-1}(x)=-2-\sqrt{x+7}$

8 a $x \in \boldsymbol{R}, x \neq 2; y \in \boldsymbol{R}, y \neq 0$

b $f^{-1}(x)=2+\frac{1}{x}$

c $x \in \boldsymbol{R}, x \neq 0; y \in \boldsymbol{R}, y \neq 2$

9 $f^{-1}(x)=2+\sqrt{x+4}$ when $x \geq 2$

10 $x \leq -3$, $f^{-1}(x)=-3-\sqrt{x+10}$

3.04

1 a 4 b 1 c −11

d 12 e −9 f 10

g 0 h 1 i −5

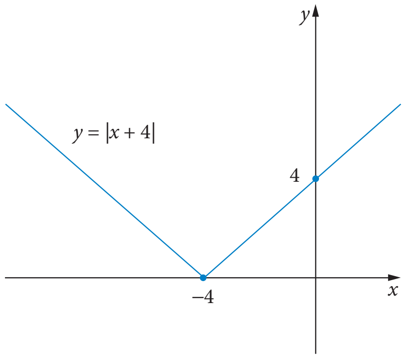
j 11

2 a $x=-2, 8$ b $x=-4, 1$ c no solution

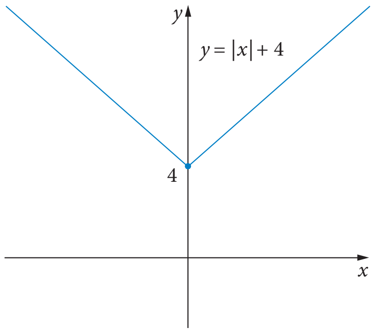
d $x=-5, 3$ e $x=\pm\frac{1}{4}$

3 $x=-2, \frac{4}{3}$

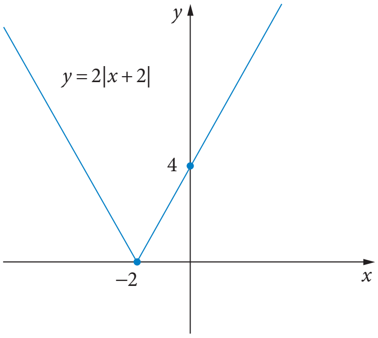
4 a $x \in \boldsymbol{R}, y \geq 0$



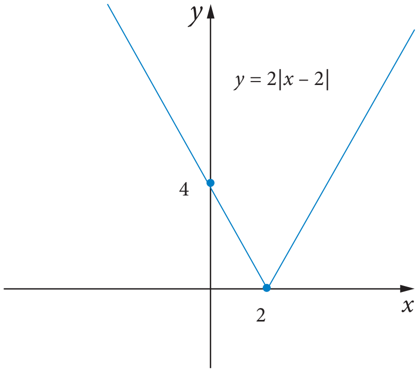
b $x \in \boldsymbol{R}, y \geq 4$



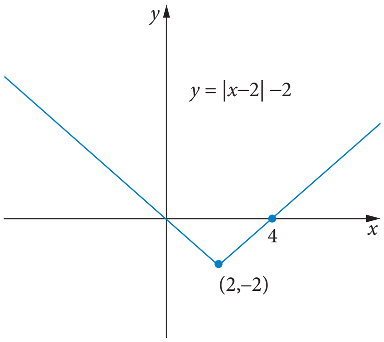
c $x \in \mathbf{R}, y \geq 0$



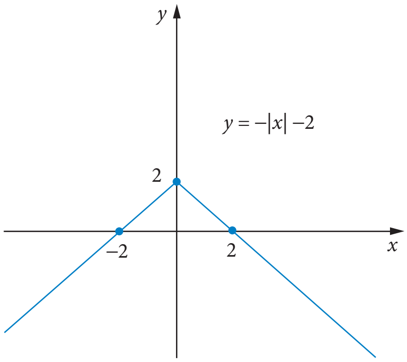
d $x \in \mathbf{R}, y \geq 0$



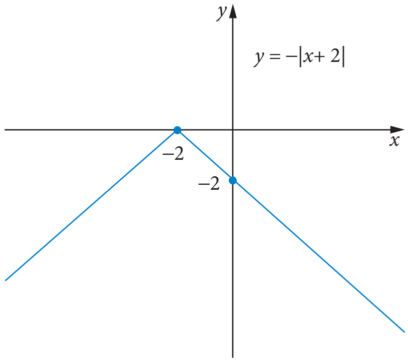
e $x \in \mathbf{R}, y \geq -2$



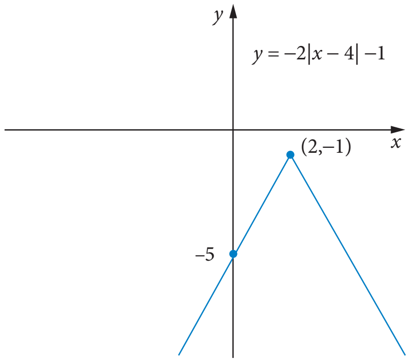
f $x \in \mathbf{R}, y \leq 2$



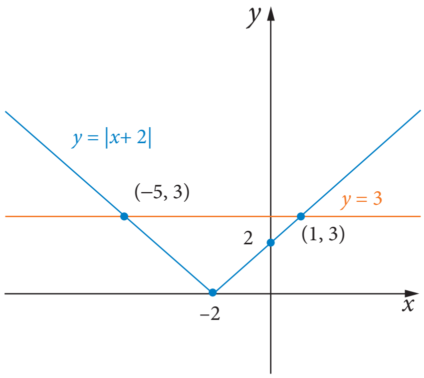
g $x \in \mathbf{R}, y \leq 0$



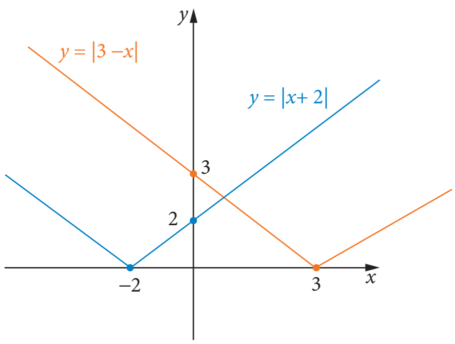
h $x \in \mathbf{R}, y \leq -1$



5 $x = -5, 1$

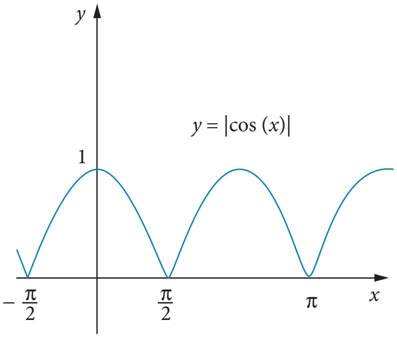


6 $x = \frac{1}{2}$

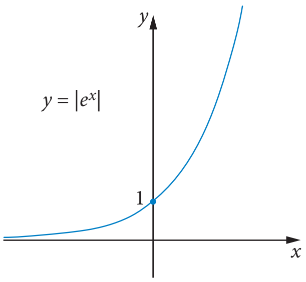


9780170250306

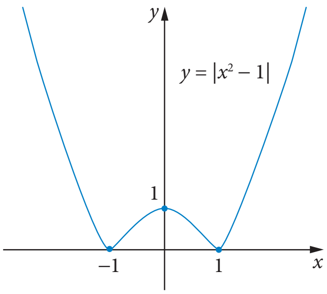
7 a $x \in \boldsymbol{R}, 0 \le y \le 1$



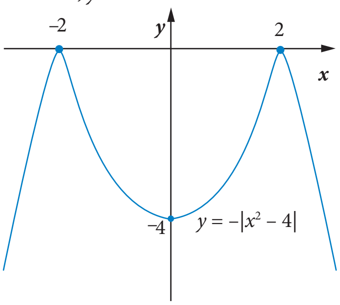
b $x \in \boldsymbol{R}, y > 0$



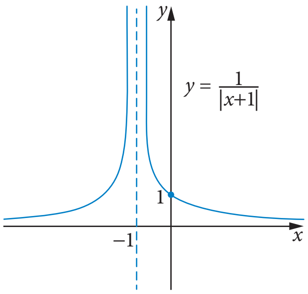
c $x \in \boldsymbol{R}, y \ge 0$



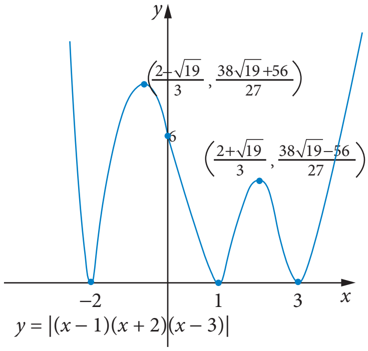
d $x \in \boldsymbol{R}, y \le 0$



e $x \in \boldsymbol{R}(x \ne -1), y > 0$



f $x \in \boldsymbol{R}, y \ge 0$



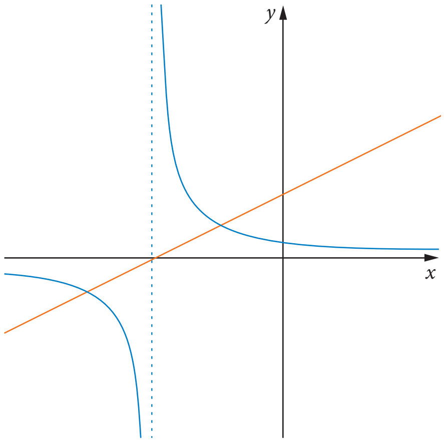
8 $x = 0, x \le -2\sqrt{2}, x \ge 2\sqrt{2}$

9 $x \le \frac{2}{3}, x \ge 1\frac{1}{3}$

10 $-2\frac{1}{2} < x < 2\frac{1}{2}$

3.05

1 a



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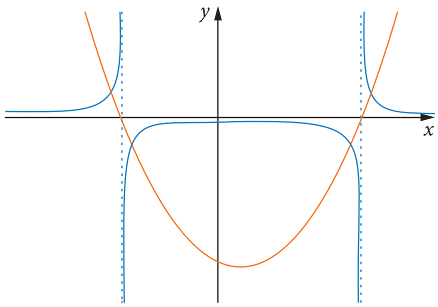

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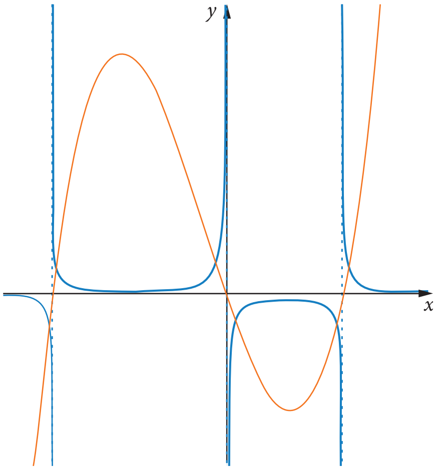

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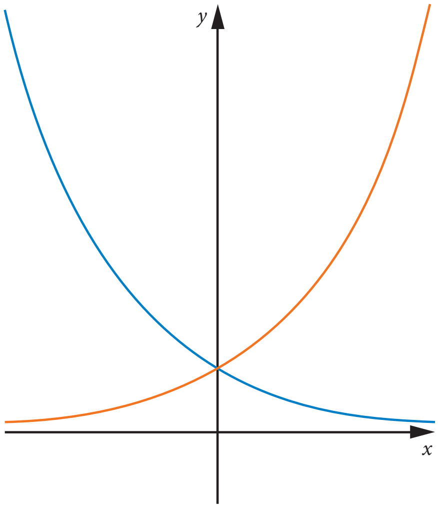
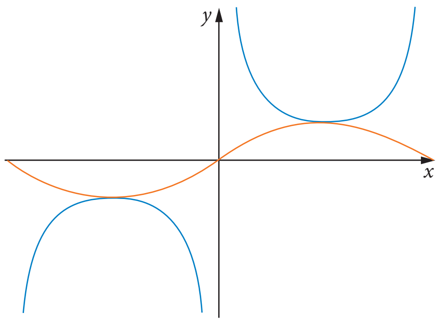
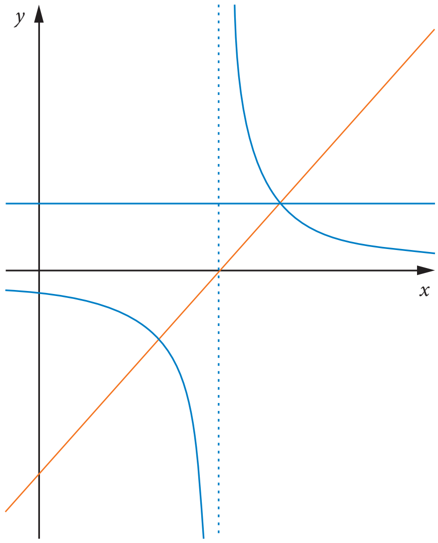


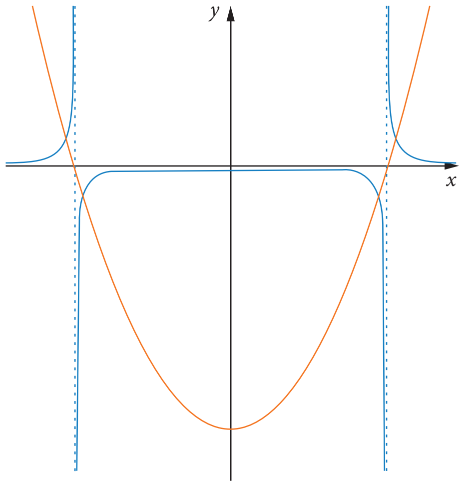
e



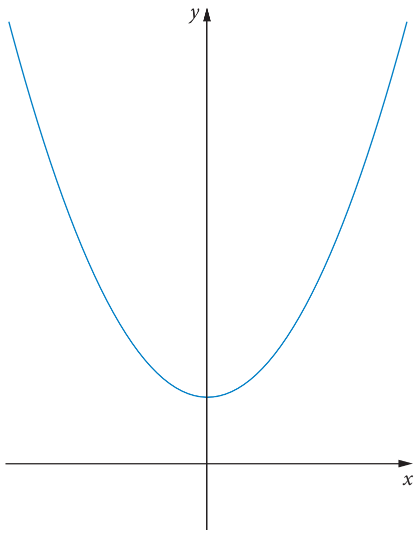
2 $1\frac{1}{2} < x \le 2$



3 $x < -3, x > 3$



4 a



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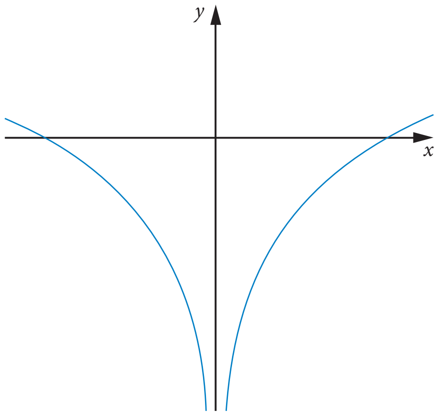

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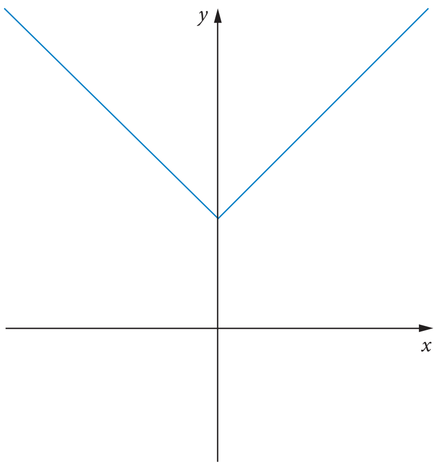

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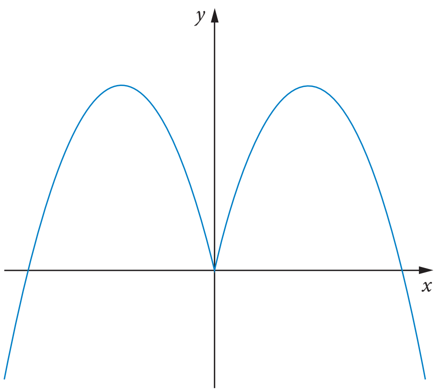

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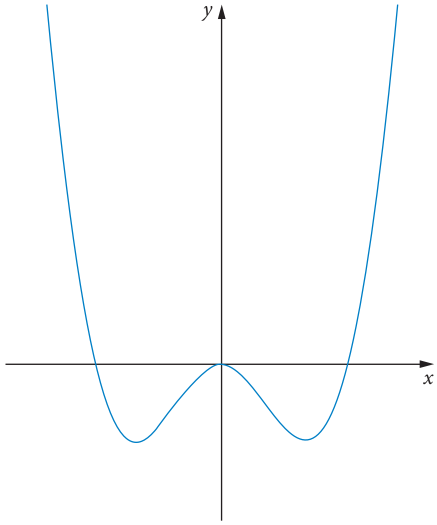
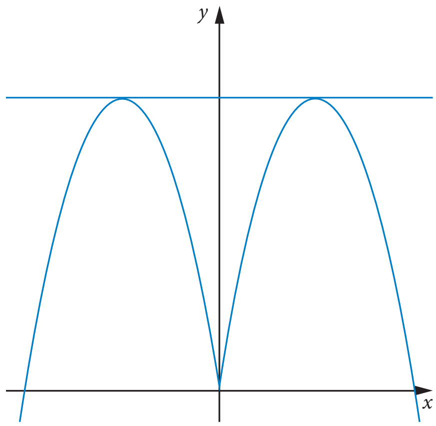
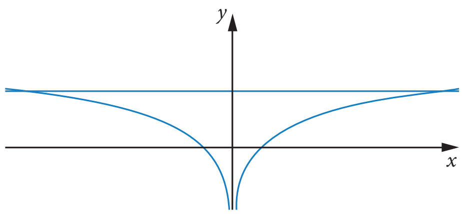


5 $x = \pm 3$



6 $-e^2 < x < e^2$



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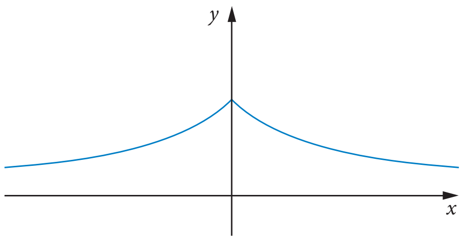

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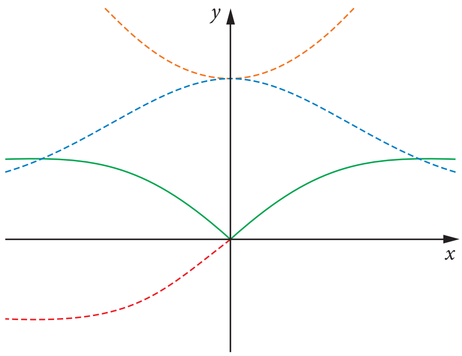
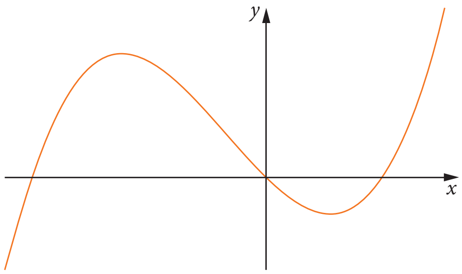
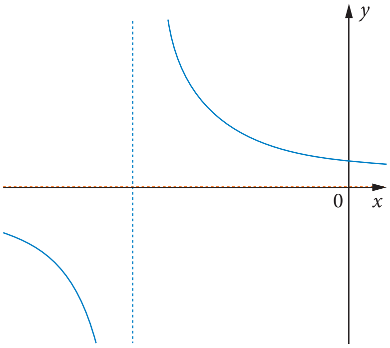


9 $x < -4, 0 < x < 2$



3.06

1 a



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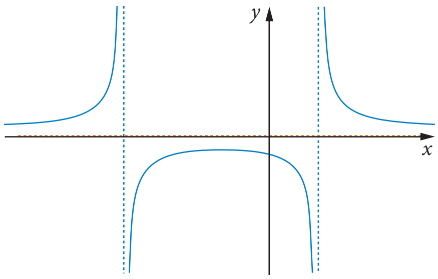

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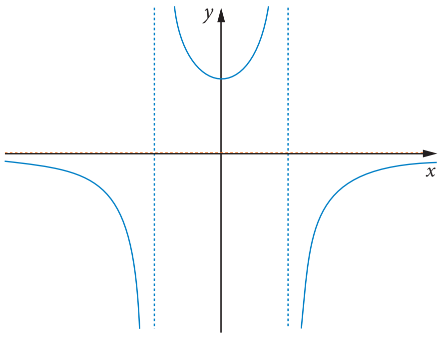

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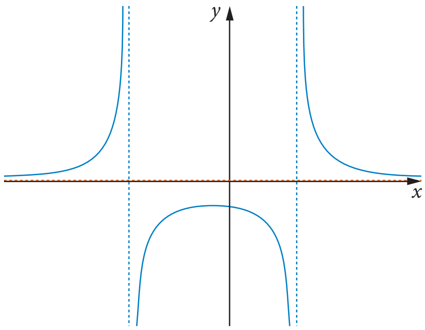

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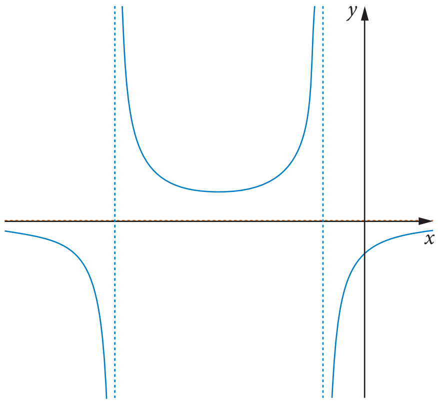

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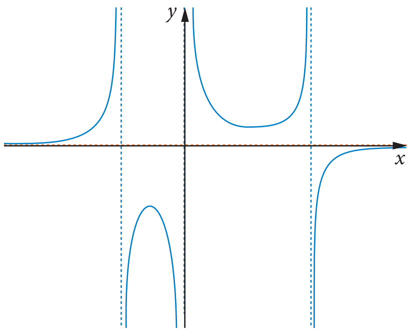

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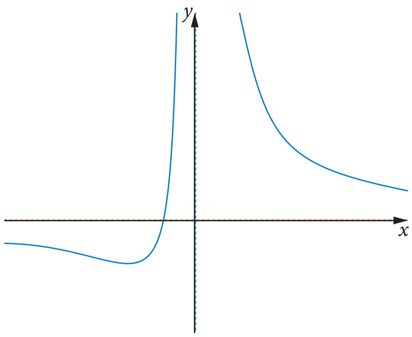
2 a



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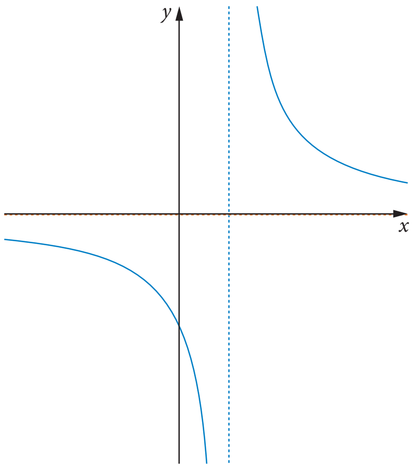

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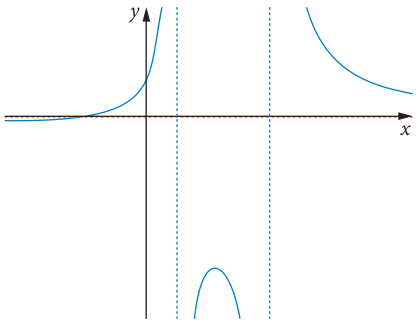

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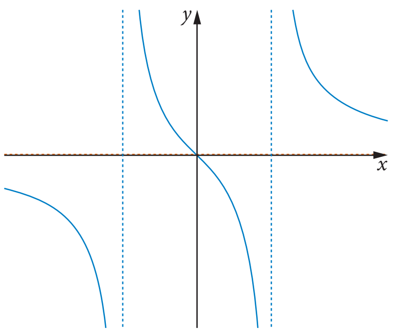

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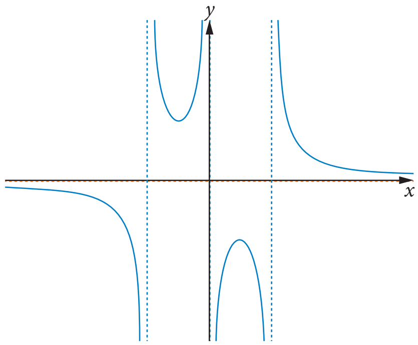

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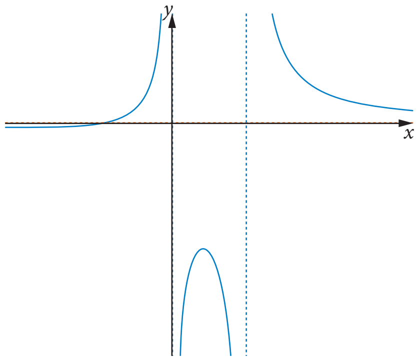

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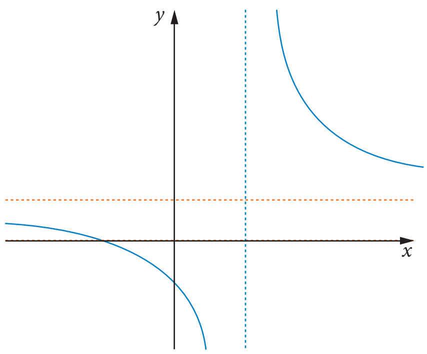

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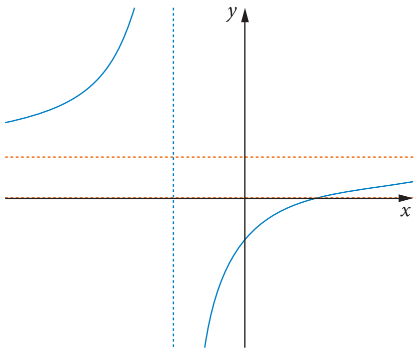

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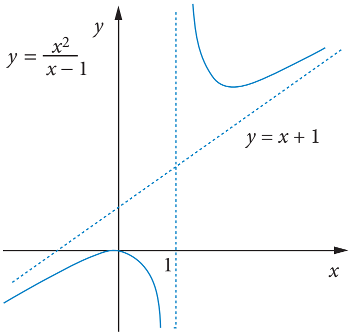

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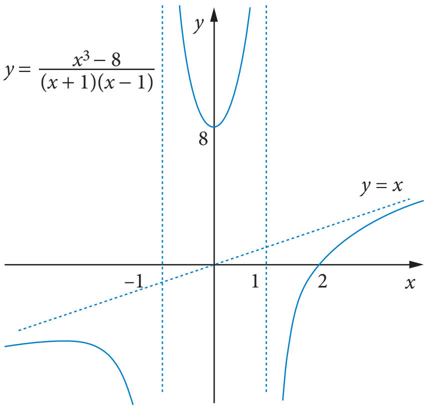
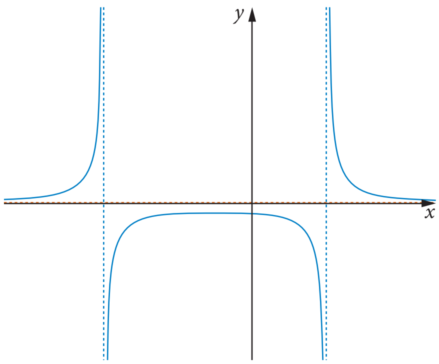
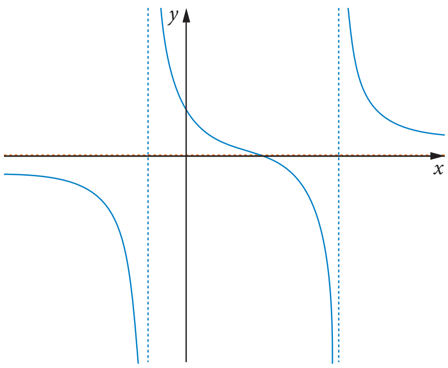


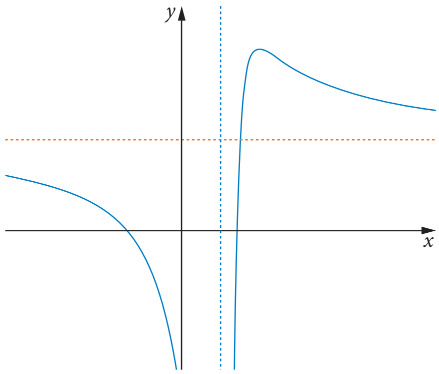
4 $x < -4, x > 2$



5 $x < -1, 1 \le x < 4$

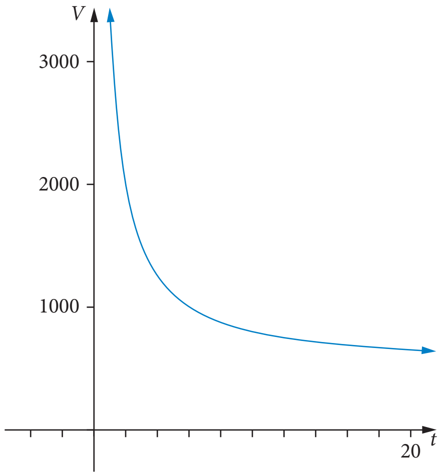


6 $-\sqrt{2} \le x < 1, 1 < x \le \sqrt{2}$



7 a $V(2) = \$2000$ **b** $V(10) = \$800$

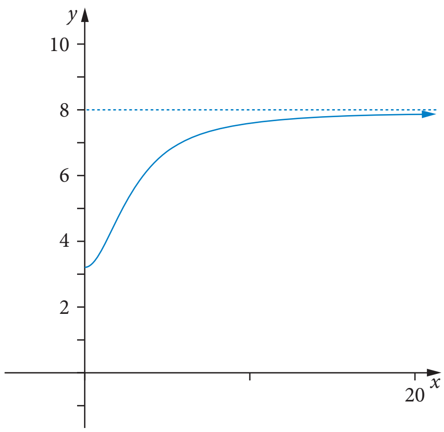
c



8 a $t = 3.06$ years, late January in 1993

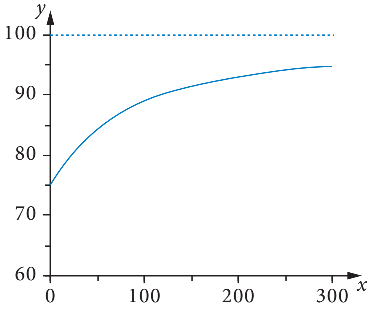
b As $t \to \infty$, sales $\to$ 8 billion games

c



9 a $\dfrac{x+60}{x+80}\times 100\%$ b $\dfrac{x+60}{x+80}\times 100 > 90, x \ge 120$

c



CHAPTER 3 REVIEW

1 D

2 D

3 B

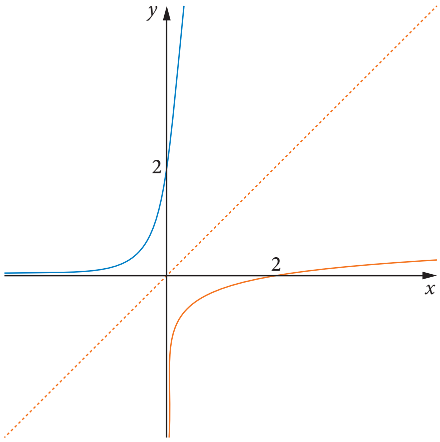
4 C

5 C

6 $x=\dfrac{2}{y}+3, f^{-1}(x)=3+\dfrac{2}{x}$

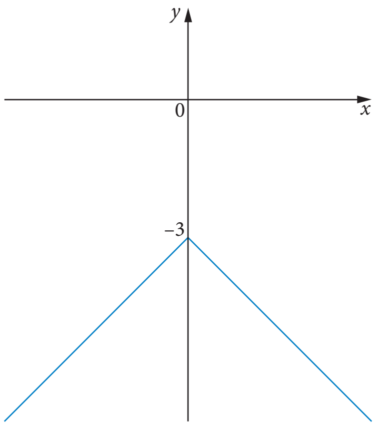
7 $x=\sqrt{5-y}, y=5-x^2$

8

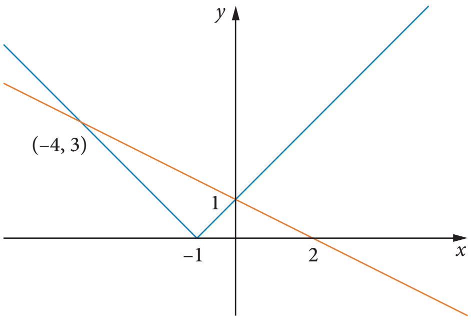


9 a −2 b 13 c −12

10 Domain: $x \in \boldsymbol{R}$, Range: $y \le -3$



11 $x = 0, -4$



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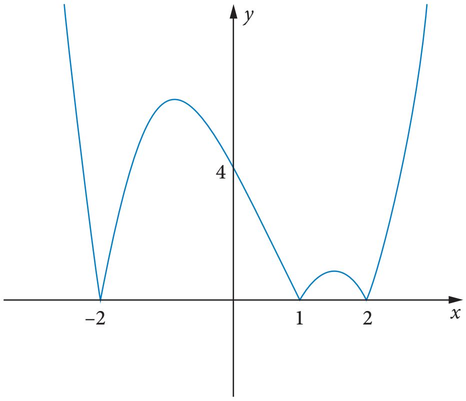

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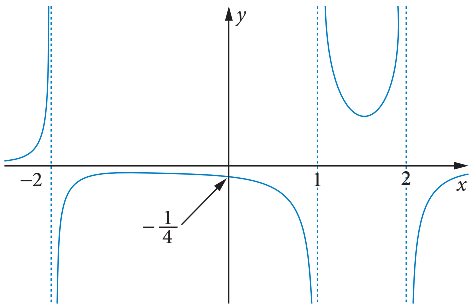

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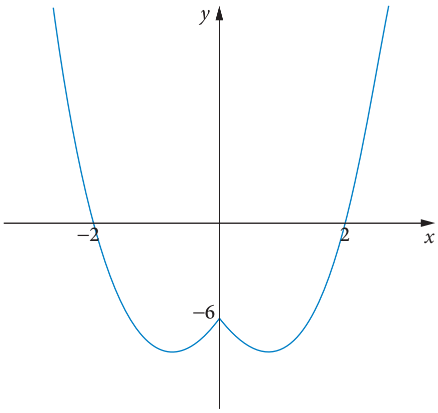

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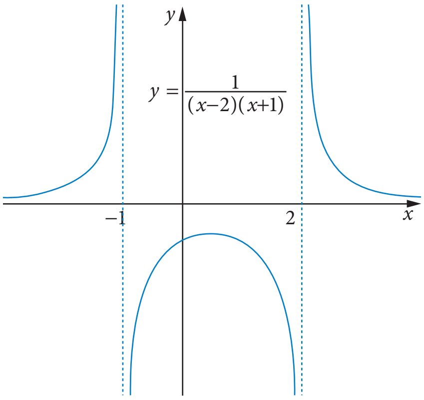

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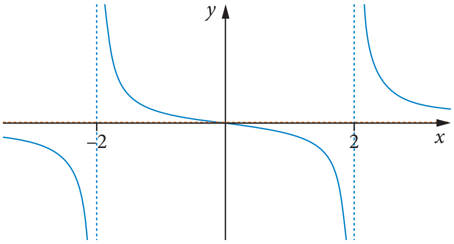

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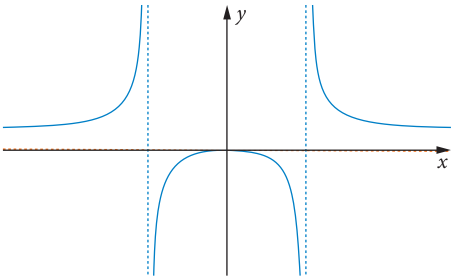
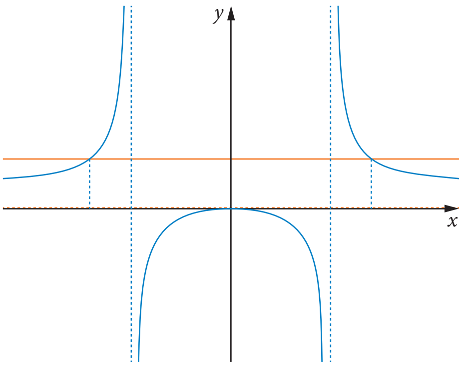


18 $(y-x)(x+2)=c$, $(y-x)(x+2)=4$

19 $x \le -2\sqrt{2}, -2 < x < 2, x \ge 2\sqrt{2}$



20 Rate $= \dfrac{100(x+15)}{x+17}$. Ross must submit the next 23 assignments to increase his return rate to 95%.

MIXED REVISION 1

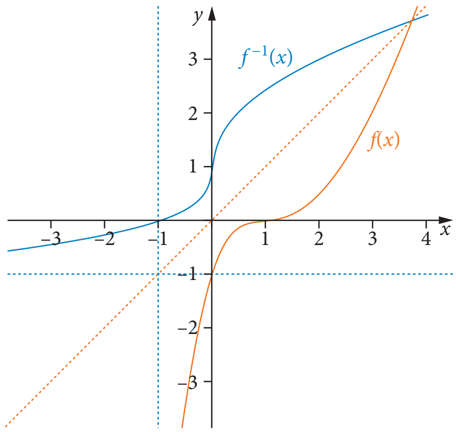
Multiple choice

1 B
2 A
3 D
4 E
5 C
6 D
7 A
8 D
9 D

Short answer

1 $16+16i\sqrt{3}$

2



3 a 7 b 26 c 24 d −6
e −6 f −25 g 33 h −51
i −51 j 33

4 a 20.07 sq.u. b 8.73 sq.u.

5 $\dfrac{x}{x-2} > -1$, $x < 1$, $x > 2$

6 $w = 7\left[\cos\left(\dfrac{\pi}{4}\right) + i\sin\left(\dfrac{\pi}{4}\right)\right]$

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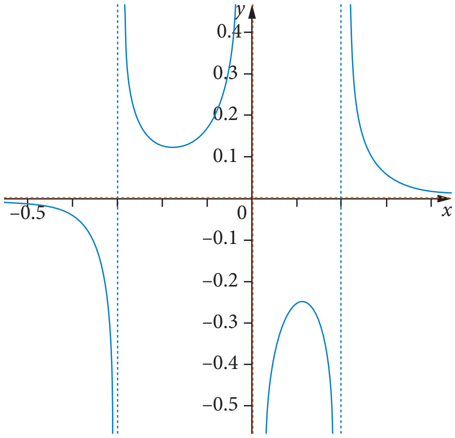
Application

1 Proof. Use $z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$ and $(\bar{z})^n = r^n\left[\cos(\theta) - i\sin(\theta)\right]^n = r^n\left[\cos(-\theta) + i\sin(-\theta)\right]^n$.

2 $\dfrac{1}{x(x-2)(x+3)} > 0$



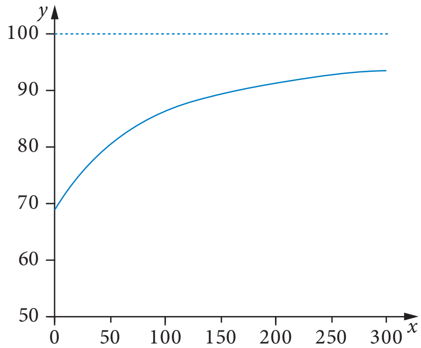
$-3 < x < 0,\ x > 2$

3 Proof

4 $z = \sqrt{2}\,\text{cis}\left(-\dfrac{\pi}{12}\right),\ z^6 = \left[\sqrt{2}\,\text{cis}\left(-\dfrac{\pi}{12}\right)\right]^6 = 8\,\text{cis}\left(-\dfrac{\pi}{2}\right)$

so $n = 6$.

5 $\dfrac{100(50+x)}{75+x} = 90$



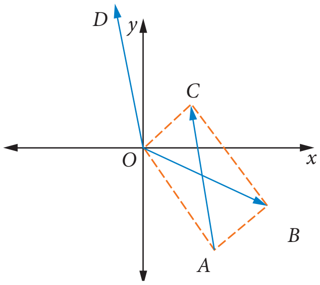
$x = 175$

6 Proof

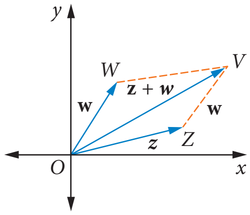
4.01

1 a see diagram b $b = c + a$

c $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ d see diagram



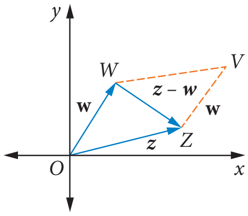
2 a



The point V represents the sum $z + w$.

b The sum of the lengths of 2 sides of a triangle is greater than the length of the 3rd side.

3



$|z-w| + |w| \geq |z|$. Equality holds when WZ and V are collinear.

4 i a $\overrightarrow{AB} = \mathbf{z}_2 - \mathbf{z}_1$ b $\overrightarrow{CB} = \mathbf{z}_2 - \mathbf{z}_3$

c $\overrightarrow{AC} = \mathbf{z}_3 - \mathbf{z}_1$

ii $w = z_4 + \dfrac{1}{2}(z_2 - z_4)$ (many other answers)

5 i a $\overrightarrow{AB} = \mathbf{w} - \mathbf{z}$ b $\overrightarrow{CB} = \mathbf{w} - \mathbf{u}$

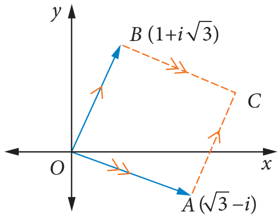
c $\overrightarrow{AC} = \mathbf{u} - \mathbf{z}$

ii a $u = v + w - z$

b $m = \dfrac{1}{2}(u+z)$ or $m = \dfrac{1}{2}(v+w)$

6 Parallelogram, opposite sides parallel and equal in length

7 a



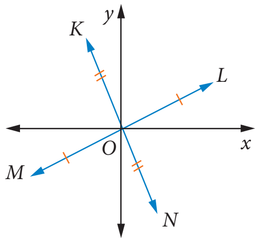
b $c = \left(\sqrt{3}+1\right) + i\left(\sqrt{3}-1\right)$ c Proof

d Proof e Square

8 a $|a| = 2\sqrt{13}, |b| = \sqrt{13}, |a-b| = \sqrt{65}$

b Proof c Right-angled

9



parallelogram since diagonals bisect each other.

10 a $c = \frac{5}{2}(1+\sqrt{3}) + \frac{5}{2}(1+\sqrt{3})i$

b $12\frac{1}{2}$ square units.

4.02

1 a $\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)$

b $\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)$

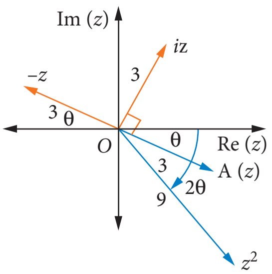
c $2\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right]$

d $\sqrt{3}\left[\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right]$

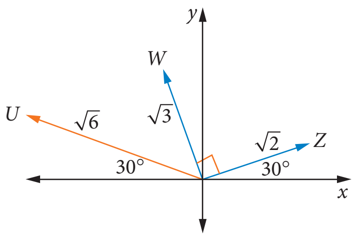
2 a i b $-i$

3 a iu b $u + iu$

4



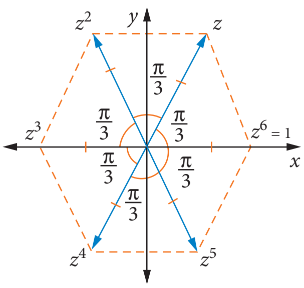
5



6 $c = 2\left[\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right]$,

$b = c + a = 2\ \text{cis}\left(\frac{7\pi}{12}\right) + \text{cis}\left(\frac{\pi}{4}\right)$

7

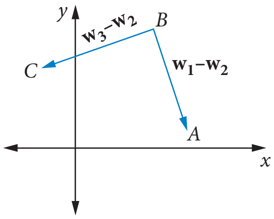


Note that connecting z_1 to z_6 forms a regular hexagon.

8 a V is $-2iz$, W is $z(1-2i)$

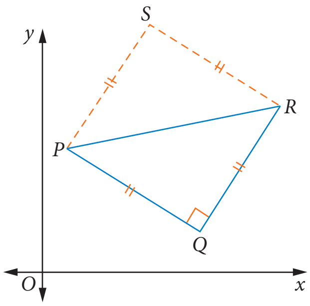
b U is $\frac{z-2iz}{2}$

9 a



b Right-angled at B.

10 a

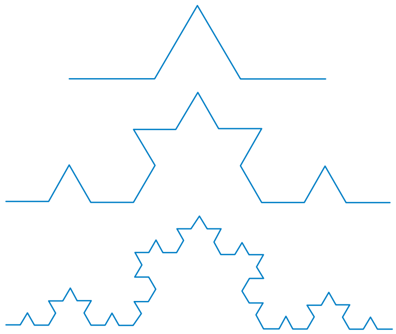


b Proof

c $PQRS$ is a square.

INVESTIGATION: THE DEVELOPMENT OF CHAOS THEORY

The Von Koch Curve is constructed starting with a line segment.
The line segment is divided into three equal line segments and an equilateral triangle constructed on the middle segment. The base of the triangle is then erased.
The process is repeated with every line segment.
Continuation produces the Koch curve.



Starting with an equilateral triangle produces a Koch snowflake.

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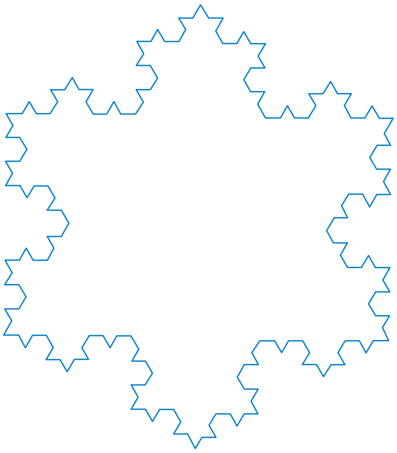
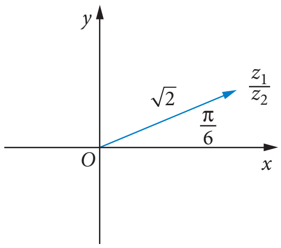


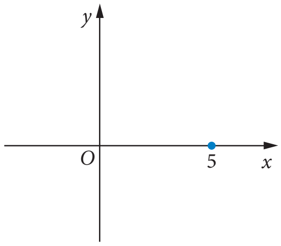
The Mandelbrot set is the set of points C in the complex plane such that the sequence $z_{n+1} = z_n^2 + C$ is bounded. Fractals are objects and sets like the Koch curve and the Mandelbrot set that are apparently chaotic but are produced by a relatively simple rule.

4.03

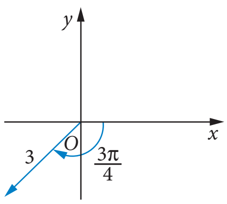
1 a $\left|\frac{z_1}{z_2}\right| = \sqrt{2}, \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{6}$



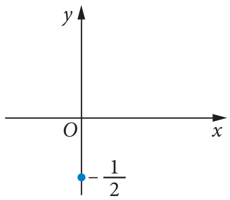
b $\left|\frac{z_1}{z_2}\right| = 5, \arg\left(\frac{z_1}{z_2}\right) = 0$



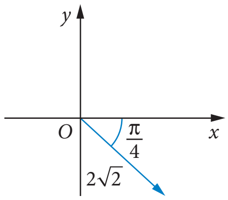
c $\left|\frac{z_1}{z_2}\right| = 3, \arg\left(\frac{z_1}{z_2}\right) = -\frac{3\pi}{4}$



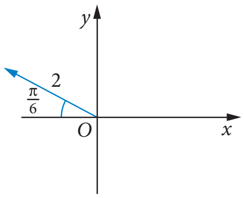
d $\left|\frac{z_1}{z_2}\right| = \frac{1}{2}, \arg\left(\frac{z_1}{z_2}\right) = -\frac{\pi}{2}$



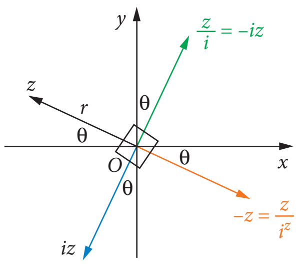
2 $\frac{z_1}{z_2} = 2\sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right)$



3 $\frac{z_1 z_2}{z_3} = 2\ \text{cis}\left(\frac{5\pi}{6}\right)$



4

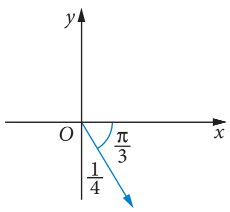


5 a $w = -iz = \frac{z}{i}$ b $w = -z$ c $w = iz$

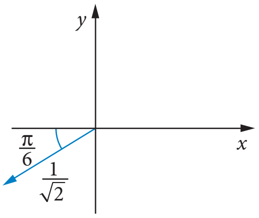
6 Hint: $|w_4 - w_1| = 2|w_2 - w_1|$,

$\arg(w_4 - w_1) - \arg(w_2 - w_1) = \frac{\pi}{2}$

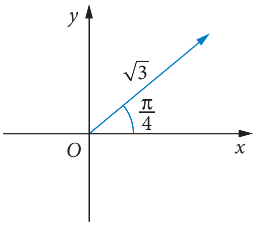
7 a



b

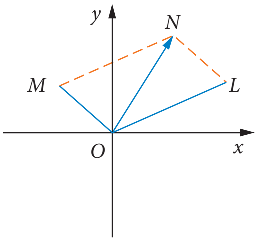


c



8 Proof

9 a

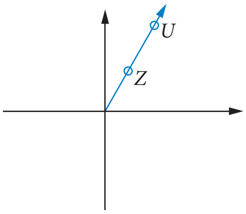


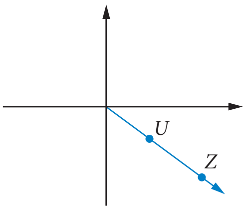
b Parallelogram c $\frac{w_2}{w_3}$ is imaginary

10 Proof

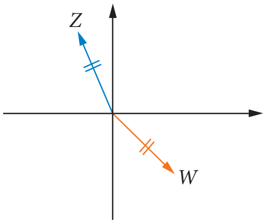
4.04

1 a z and u lie on the same line through the origin, on the same side of O.

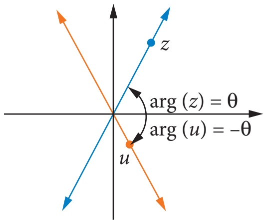




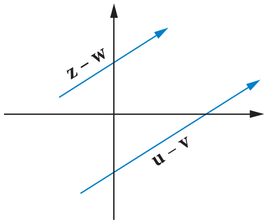
b z and w are the same distance from O.



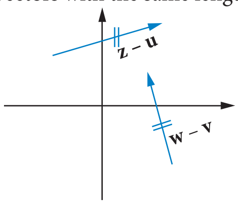
c z and u lie on lines through O that are reflections over the x-axis



d parallel vectors (same direction)



e vectors with the same length



f vectors same length and parallel

g same as (a)

h same as (c)

i diagonals of quadrilateral equal, rectangle

j $z - u = v - w$, same as (f)

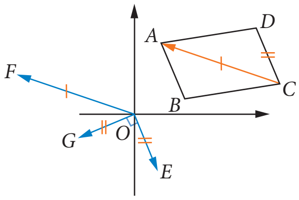
k quadrilateral has equal diagonals, perpendicular, square.

2 $\theta - \pi$

3 Proof

4 a Proof b Proof c Proof d Proof

5

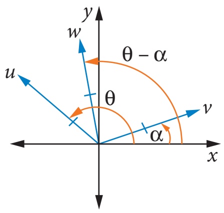


6 a Proof b Proof c Proof

7 $z = u \text{ cis}\left(\frac{\pi}{4}\right)$, $w = -2u \text{ cis}\left(\frac{\pi}{4}\right)$, $v = \overline{u}$

8 Diagrams will vary.

a $w = \frac{u}{v}$



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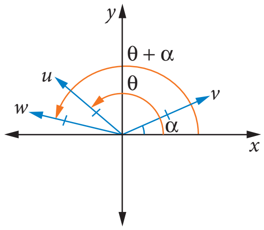
b $w = uv$

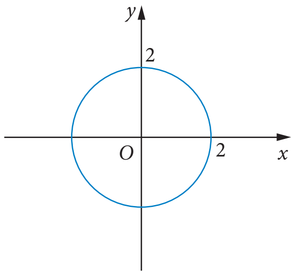


9 a $\theta = \dfrac{2\pi}{5}$

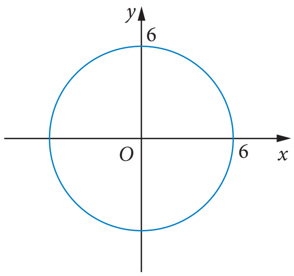
b $z_1 = \text{cis}\left(\dfrac{2\pi}{5}\right)$, $z_2 = \text{cis}\left(\dfrac{4\pi}{5}\right)$, $z_3 = \text{cis}\left(\dfrac{-4\pi}{5}\right)$, $z_4 = \text{cis}\left(\dfrac{-2\pi}{5}\right)$, $z_5 = 1$

4.05

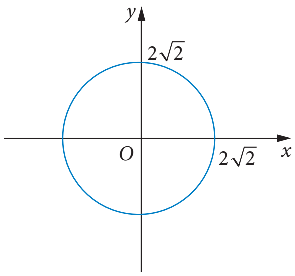
1 a



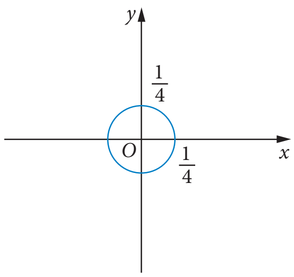
b



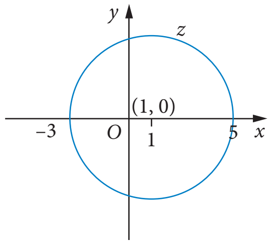
c



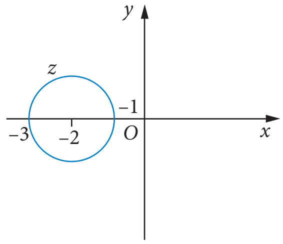
d



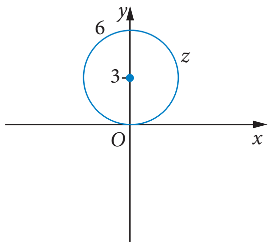
e



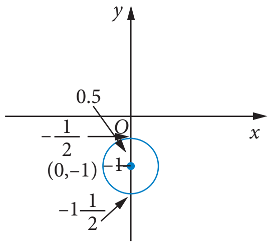
f



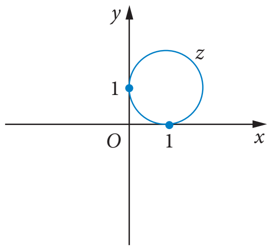
g



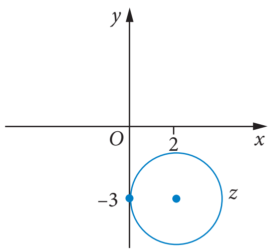
h



2 a



b



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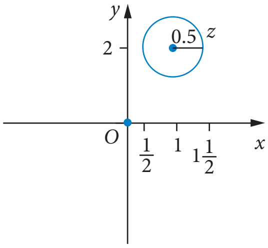

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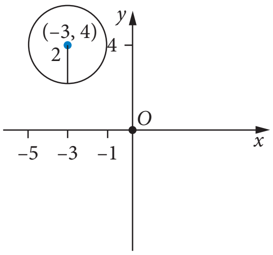
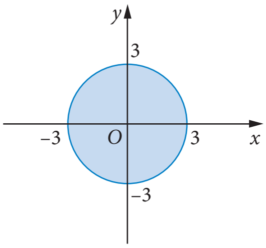
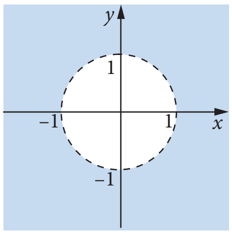


3 a



b



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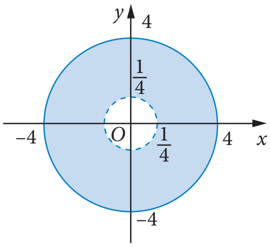
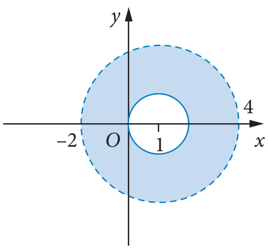
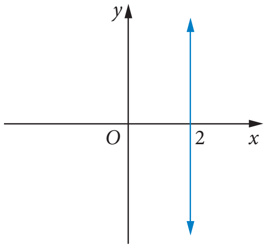


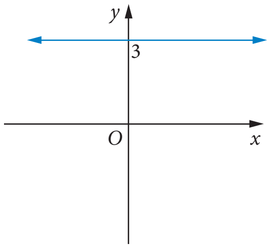
d



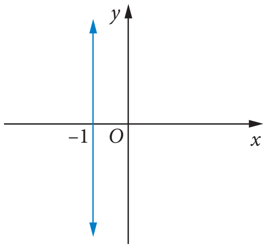
4 a



b



c



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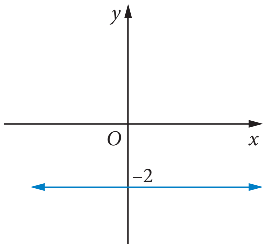
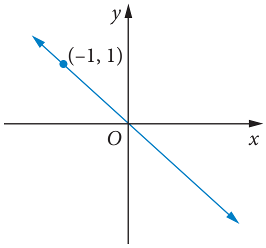
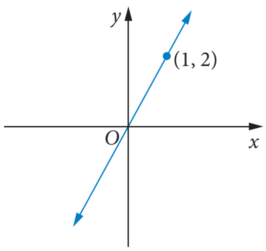


5 a $x = -y$



b $y = 2x$



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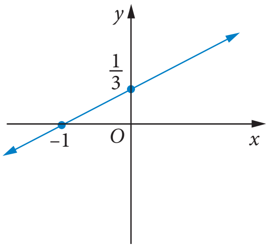
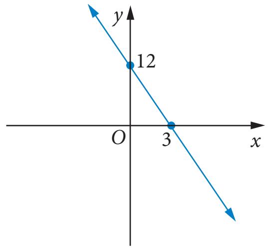
c $x = 3y - 1$



d $y + 4x = 12$



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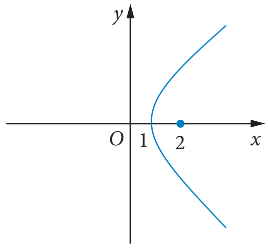

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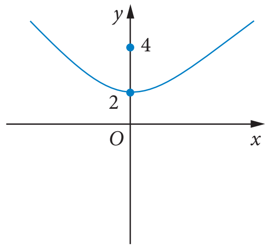

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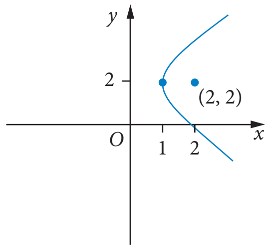

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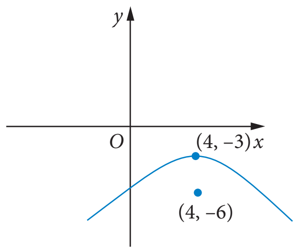

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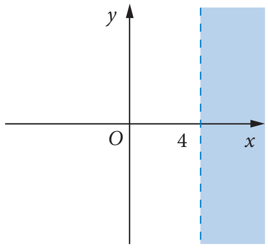

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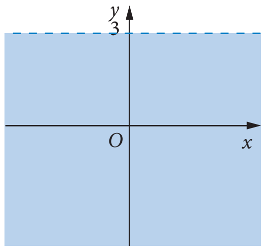

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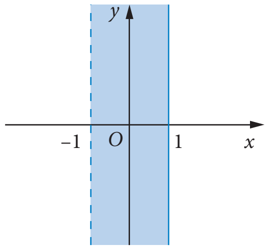

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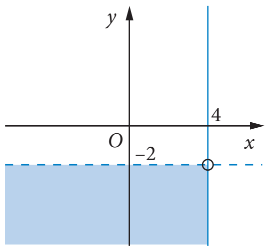

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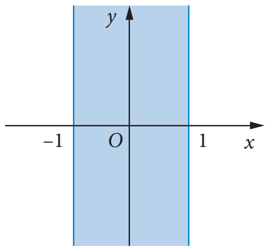

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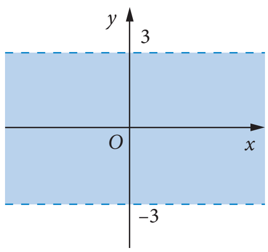

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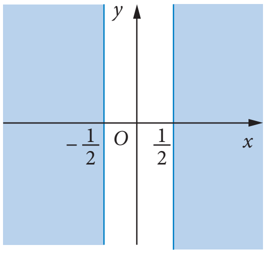

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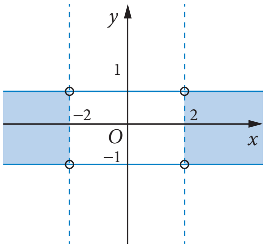
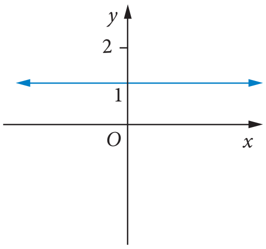
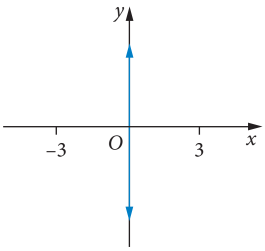


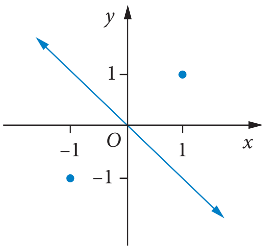
9 a A line equidistant from (0, 0) and (0, 2).



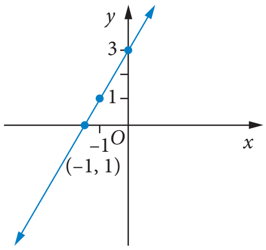
b A line equidistant from (3, 0) and (−3, 0).



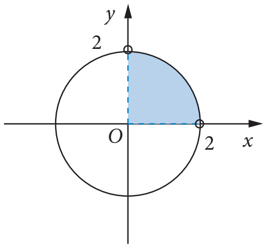
c A line equidistant from (1, 1) and (−1, −1).



d A line equidistant from (3, −1) and (−5, 3).



10 a



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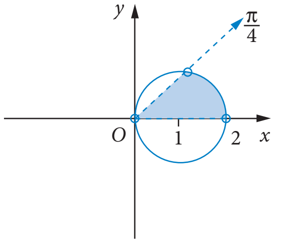

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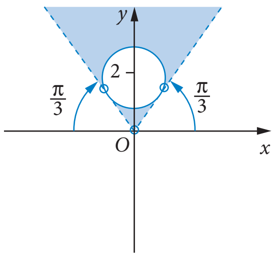

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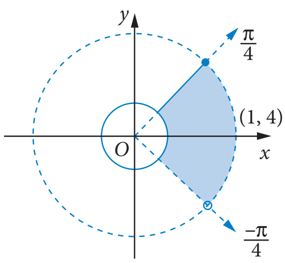
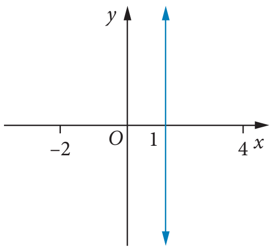


11 a



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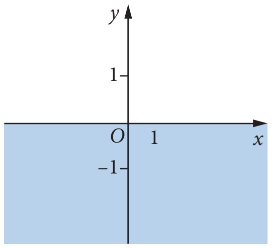
b



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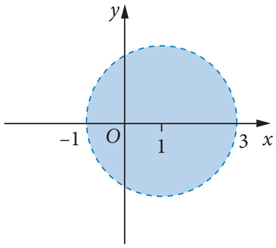
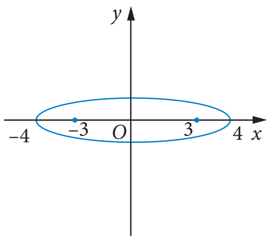


12 Centre (−5, 0), radius $4\sqrt{2}$ units

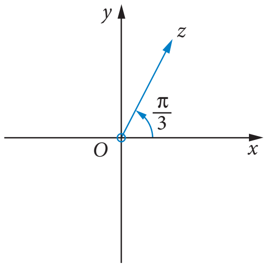
13



14 $\frac{\pi}{6}$

4.06

1 a



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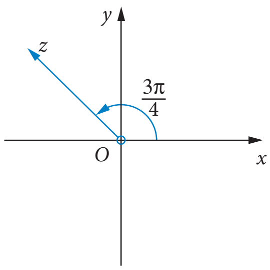

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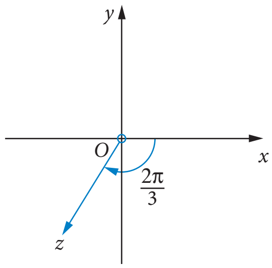

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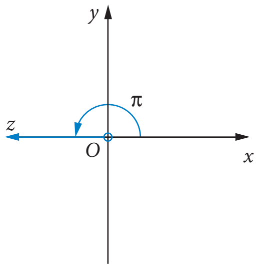

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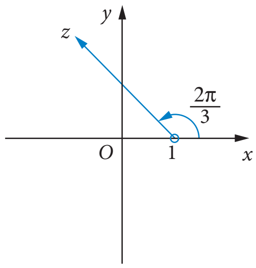

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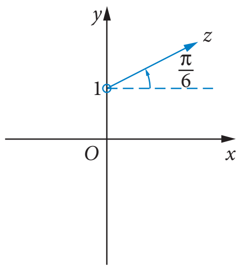

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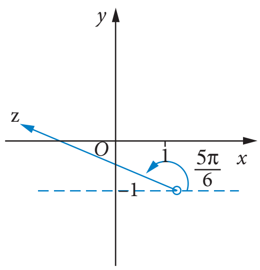

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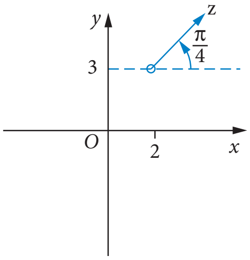

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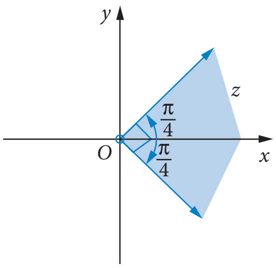

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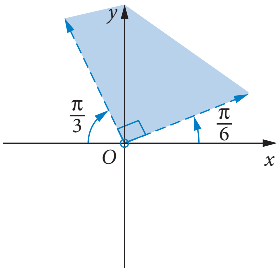

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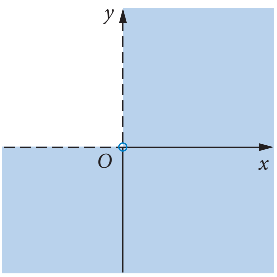

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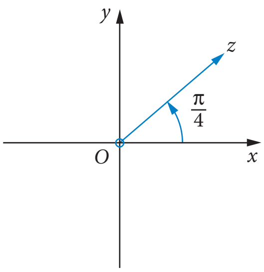

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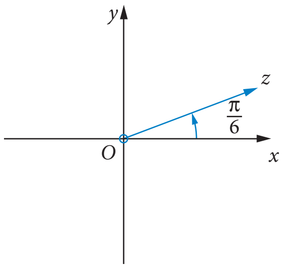

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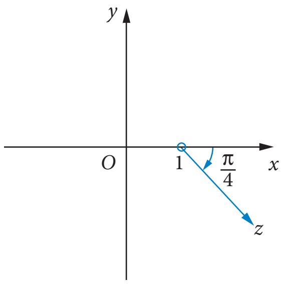

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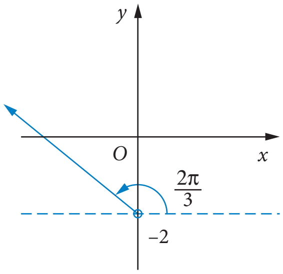

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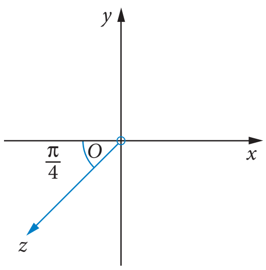

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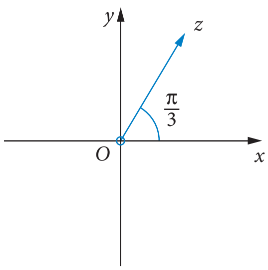

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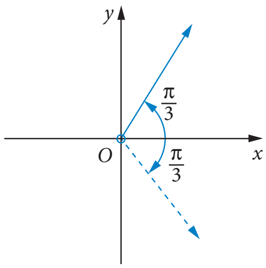

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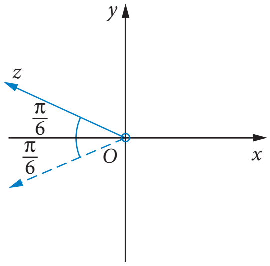

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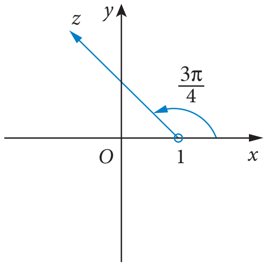

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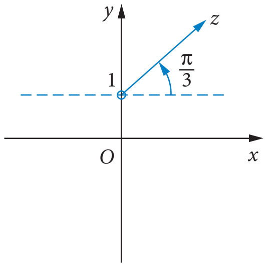

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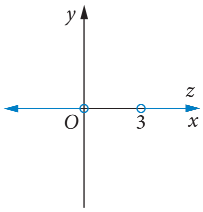

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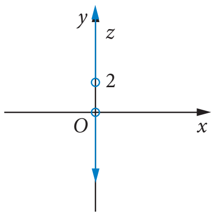

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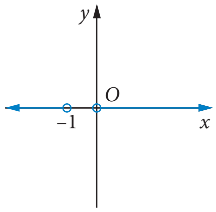

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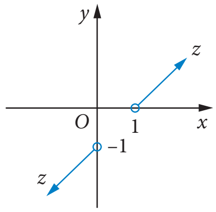

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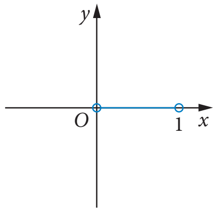

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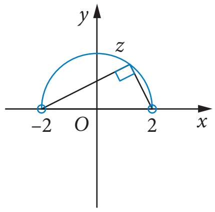

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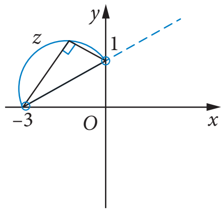

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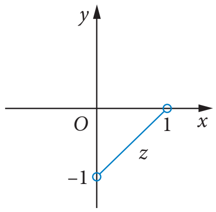

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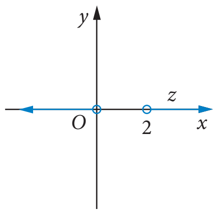

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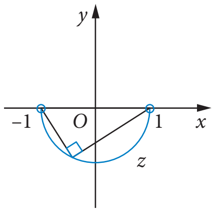

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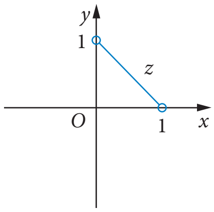

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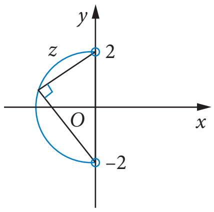

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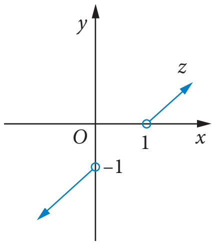

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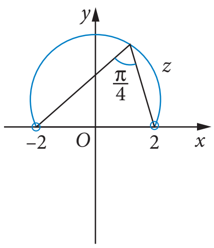

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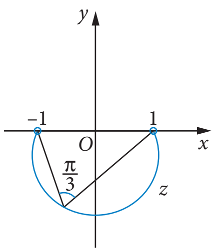

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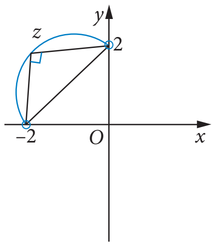
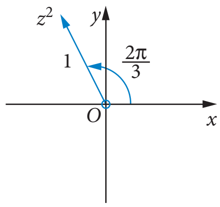


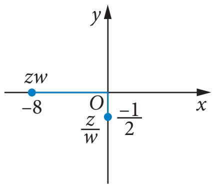
CHAPTER 4 REVIEW

1 B
2 C
3 D
4 B
5 E
6

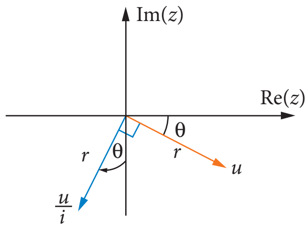


7 a $zw = -8$, $\text{Im}(zw) = 0$ **b** $\frac{z}{w} = -\frac{i}{2}, \text{Re}\left(\frac{z}{w}\right) = 0$

c

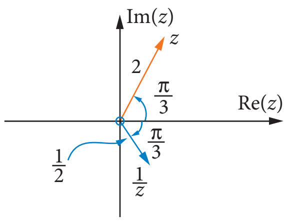


8



9 a $\text{mod}\left(\frac{1}{z}\right) = \frac{1}{2}, \arg\left(\frac{1}{z}\right) = -\frac{\pi}{3}$

b

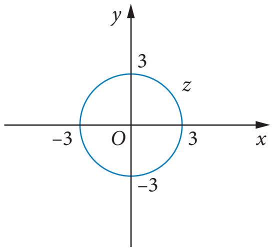


10 Proof
11 a Rectangle **b** Parallelogram
c Trapezium

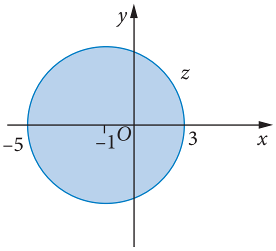
9780170250306

12 $z_2 = -\overline{z_1}$, $z_3 = -z_1$, $z_4 = \overline{z_1}$

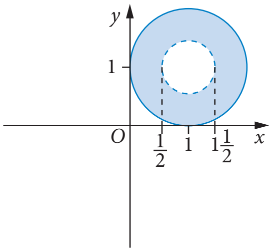
13 a $x^2 + y^2 = 9$



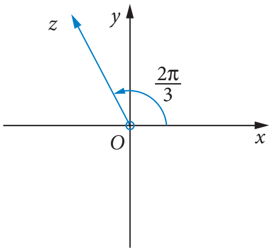
b $(x+1)^2 + y^2 \le 16$



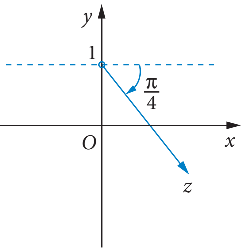
c $\frac{1}{4} < (x-1)^2 + (y-1)^2 \le 1$



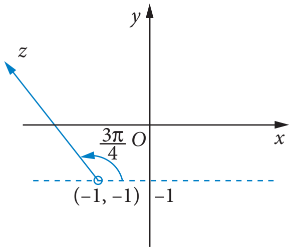
14 a



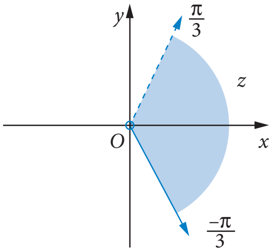
b



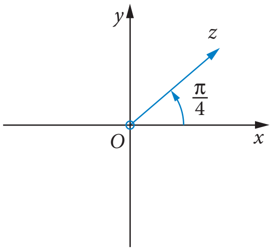
c



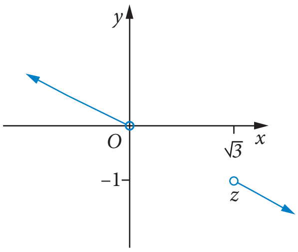
15 a



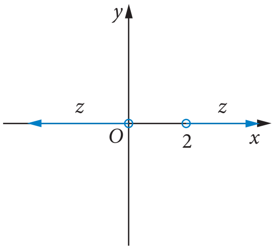
b



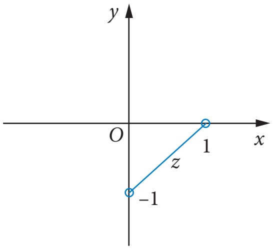
c



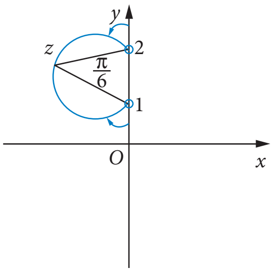
16 a



b



c

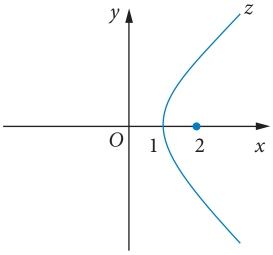


17 a Parabola $V(1, 0)$ focus $(2, 0)$

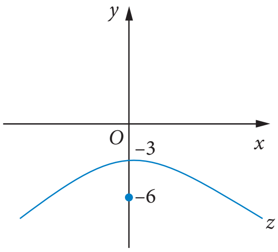
b Parabola $V(0, -3)$ focus $(0, -6)$

c Line through $(-1, 0)$ and $(0, -1)$

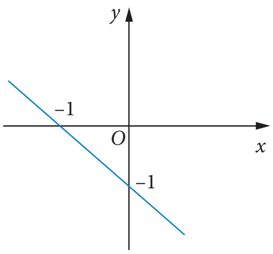
a



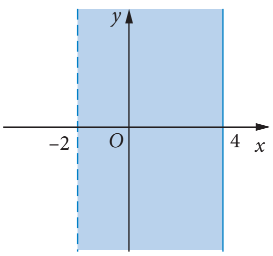
b



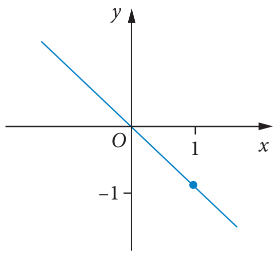
c



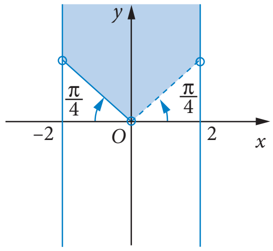
18 a



b



c



5.01

1 D

2 B

3 C

4 C

5 a $y = 3x$ b $x = 5$ c $y = 6$

d $y = 7 - x$ e $x = \frac{1}{4}(5 - y^2)$ f $y = (x + 1)^3 + 6$

g $y = \left(\frac{1}{x-2}\right)^2 + 1$ h $y = \frac{x}{1-x}$

6 a Domain: $x \in \boldsymbol{R}$, Range: $y \in \boldsymbol{R}$

b Domain: $x = 5$, Range: $y \in \boldsymbol{R}$

c Domain: $x \in \boldsymbol{R}$, Range: $y = 6$

d Domain: $x \in \boldsymbol{R}$, Range: $y \in \boldsymbol{R}$

e Domain: $x \geq 0$, Range: $y \in \boldsymbol{R}$

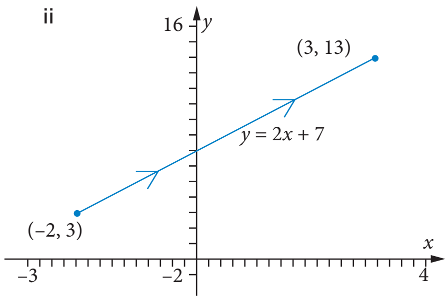
f Domain: $x \in \boldsymbol{R}$, Range: $y \in \boldsymbol{R}$

g Domain: $x > 2$, Range: $(1, \infty)$

h Domain: $(0, 1)$, Range: $(0, \infty)$

7 a i $y = 2x + 7$

ii



iii Domain: $-2 \leq x \leq 3$; Range: $3 \leq y \leq 13$

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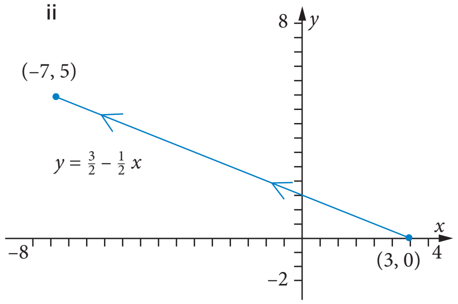
b i $y = \frac{3}{2} - \frac{1}{2}x$

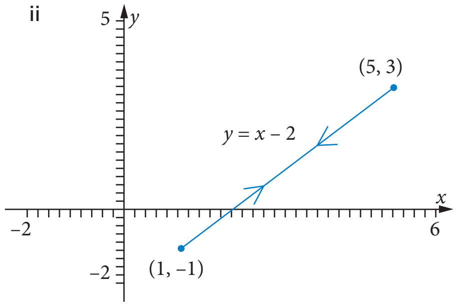
ii



iii Domain: $-7 \le x \le 3$; Range: $0 \le y \le 5$

c i $y = x - 2$

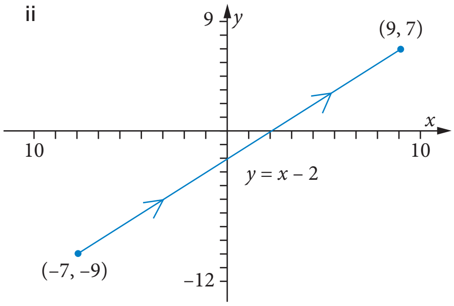
ii



iii Domain: $1 \le x \le 5$; Range: $-1 \le y \le 3$

d i $y = x - 2$

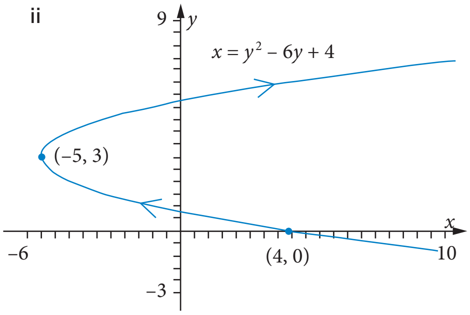
ii



iii Domain: $-7 \le x \le 9$; Range: $-9 \le y \le 7$

e i $x = y^2 - 6y + 4$

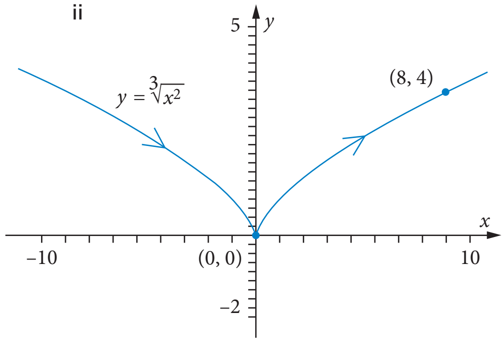
ii



iii Domain: $x \ge -5$; Range: $y \in \boldsymbol{R}$

f i $y = \sqrt[3]{x^2}$

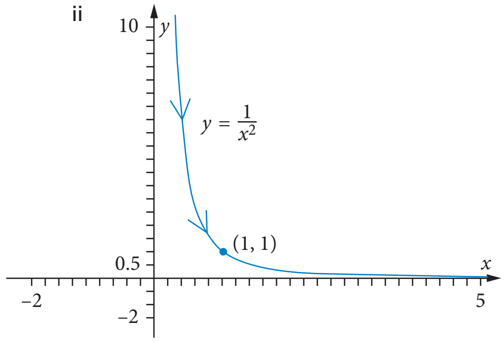
ii



iii Domain: $x \in \boldsymbol{R}$; Range: $y \ge 0$

g i $y = \frac{1}{x^2}$

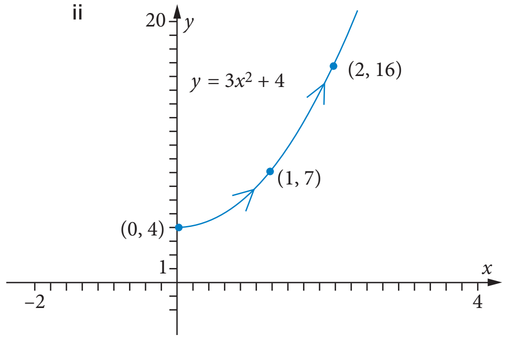
ii



iii Domain: $x > 0$; Range: $y \in \boldsymbol{R}^+$

h i $y = 3x^2 + 4$

ii



iii Domain: $x \ge 0$; Range: $y \ge 4$

8 a $x^2 + y^2 = 1$ b $x^2 + y^2 = 1$

c $x^2 + y^2 = 1$ d $\frac{x^2}{9} + \frac{y^2}{4} = 1$

e $y = 4 - \frac{4}{3}x$ f $\frac{x^2}{4} - \frac{y^2}{9} = 1$

g $x^2 + y^2 = 9$ h $y = \frac{3}{2}x - 3$

9 a Domain: $[-1, 1]$, Range: $[-1, 1]$

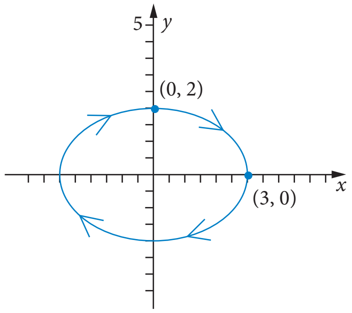
b Domain: $[0, 1]$, Range: $[-1, 1]$

c Domain: $[-1, 0]$, Range: $[-1, 0]$

d Domain: [−3, 3], Range: [−2, 2]
e Domain: [0, 3], Range: [0, 4]
f Domain: [2, ∞), Range: [0, ∞)
g Domain: [−3, 3], Range: [0, 3]
h Domain: [2, ∞), Range: [0, ∞)

10 a i $\frac{x^2}{9}+\frac{y^2}{4}=1$

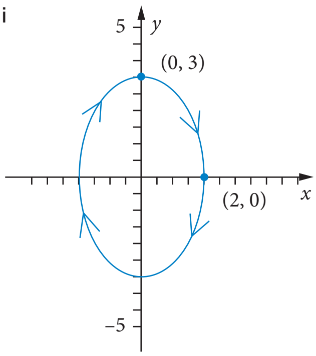
ii



iii Domain: [−3, 3]; Range: [−2, 2]

b i $\frac{x^2}{4}+\frac{y^2}{9}=1$

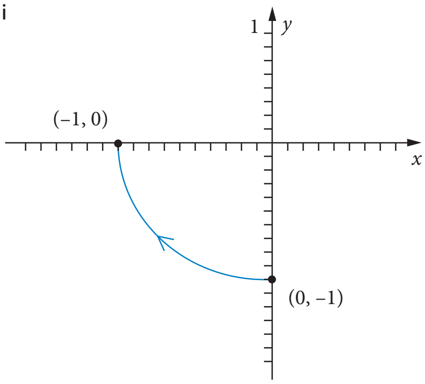
ii



iii Domain: [−2, 2]; Range: [−3, 3]

c i $x^2+y^2=1$

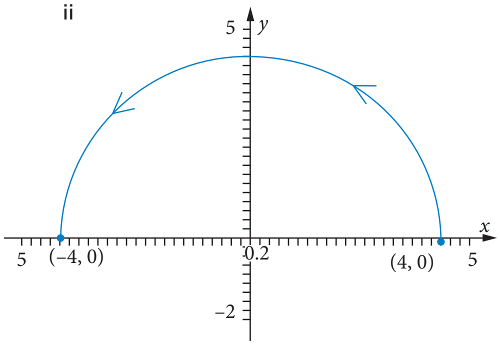
ii



iii Domain: [−1, 0]; Range: [−1, 0]

d i $x^2+y^2=16$

ii



iii Domain: [−4, 4]; Range: [0, 4]

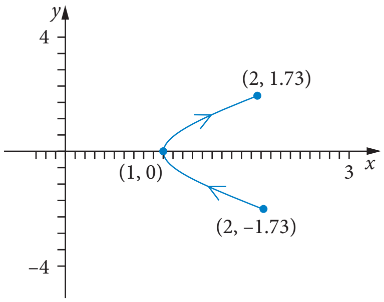
11 a (−4, 7) b 9

12 a $\sqrt{17}$ b $t=\frac{2}{5}$ or 1

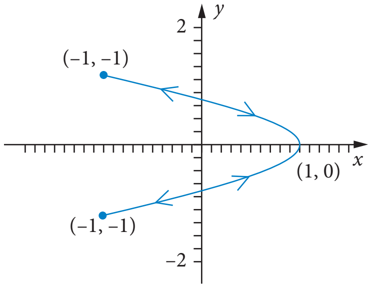
13 a $\sqrt{5}$ b $t=2\frac{1}{2}$

c $(13, 17\frac{1}{2})$ and $(0, 6\frac{1}{8})$

14 a $x^2-y^2=1$: Domain: [1, 2], Range: [−1.73, 1.73]



b $x=1-2y^2$: Domain: [−1, 1], Range:[−1, 1]



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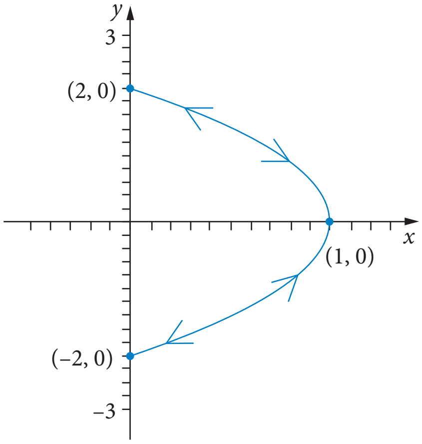
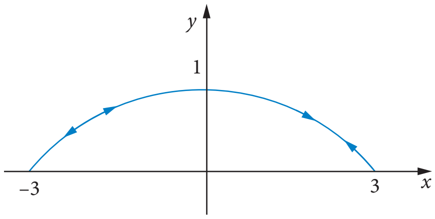
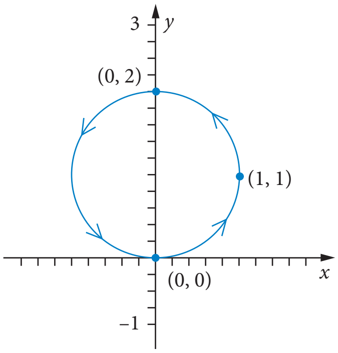
c $x = 1 - \frac{1}{4}y^2$: Domain: [0, 1], Range:[−2, 2]



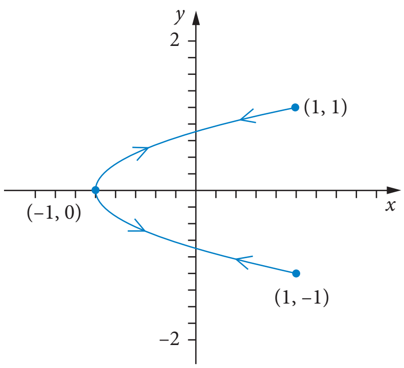
d $y = 1 - \frac{1}{9}x^2$: Domain: [−3, 3], Range:[0, 1]



e $x^2 + (y-1)^2 = 1$: Domain: [−1, 1], Range:[0, 2]



f $x = 2y^2 - 1$: Domain: [−1, 1], Range:[−1, 1]



15 The particles collide at $t = \frac{2}{3}$ at $(-7\frac{1}{3}, -\frac{25}{27})$.

5.02

1 B
2 C
3 C
4 A
5 a $\mathbf{r}(t) = 2t\mathbf{i} - 3\mathbf{j}$; $\mathbf{r}(t) = 2\mathbf{i}$
b $\mathbf{r}(t) = 10\mathbf{i} - 7\mathbf{j}$; $\mathbf{r}(t) = 0$
c $\mathbf{r}(t) = 20t^3\mathbf{i} + 8\mathbf{j}$; $\mathbf{r}(t) = 60t^2\mathbf{i}$
d $\mathbf{r}(t) = e^t\mathbf{i} + 2e^{2t}\mathbf{j}$; $\mathbf{r}(t) = e^t\mathbf{i} + 4e^{2t}\mathbf{j}$
e $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$; $\mathbf{r}(t) = -\sin(t)\mathbf{i} + \cos(t)\mathbf{j}$
f $\mathbf{r}(t) = -5\sin(t)\mathbf{i} - 5\cos(t)\mathbf{j}$;
$\mathbf{r}(t) = -5\cos(t)\mathbf{i} + 5\sin(t)\mathbf{j}$
g $\mathbf{r}(t) = -3\mathbf{i} + 2\sin(t)\mathbf{j}$; $\mathbf{r}(t) = 2\cos(t)\mathbf{j}$
h $\mathbf{r}(t) = -\sin(t)\mathbf{i} - 3\cos(t)\mathbf{j}$;
$\mathbf{r}(t) = -\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j}$
6 a $|\mathbf{r}(t)| = \sqrt{4t^2+9}$; $|\mathbf{r}(t)| = 2$
b $|\mathbf{r}(t)| = \sqrt{149}$; $|\mathbf{r}(t)| = 0$
c $|\mathbf{r}(t)| = \sqrt{400t^6+64}$; $|\mathbf{r}(t)| = 60t^2$
d $|\mathbf{r}(t)| = e^t\sqrt{4e^{2t}+1}$; $|\mathbf{r}(t)| = e^t\sqrt{16e^{2t}+1}$
e $|\mathbf{r}(t)| = 1$; $|\mathbf{r}(t)| = 1$;
f $|\mathbf{r}(t)| = 5$; $|\mathbf{r}(t)| = 5$
g $|\mathbf{r}(t)| = \sqrt{9+4\sin^2(t)}$; $|\mathbf{r}(t)| = 2|\cos(t)|$
h $|\mathbf{r}(t)| = \sqrt{1+8\cos^2(t)}$; $|\mathbf{r}(t)| = \sqrt{1+8\sin^2(t)}$
7 a $\mathbf{r}(t) = 5\mathbf{i} + 3t\mathbf{j}$; $\mathbf{r}(t) = 3\mathbf{j}$
b $\mathbf{r}(t) = 12\mathbf{i} - 10(5t-3)\mathbf{j}$; $\mathbf{r}(t) = -50\mathbf{j}$
c $\mathbf{r}(t) = -6e^{8-6t}\mathbf{i} - 42e^{-7t}\mathbf{j}$; $\mathbf{r}(t) = 36e^{8-6t}\mathbf{i} + 294e^{-7t}\mathbf{j}$
d $\mathbf{r}(t) = -e^{\frac{t}{4}+5}\mathbf{i} - 10e^{6-5t}\mathbf{j}$; $\mathbf{r}(t) = -\frac{1}{4}(e^{\frac{t}{4}+5})\mathbf{i} + 50e^{6-5t}\mathbf{j}$
e $\mathbf{r}(t) = -6(2-3t)\mathbf{i} + 3\sin(3t)\mathbf{j}$;
$\mathbf{r}(t) = 18\mathbf{i} + 9\cos(3t)\mathbf{j}$
f $\mathbf{r}(t) = \sin(2t)\mathbf{i} - \sec^2(t)\mathbf{j}$;
$\mathbf{r}(t) = 2\cos(2t)\mathbf{i} - 2\tan(t)\sec^2(t)\mathbf{j}$
8 $\mathbf{r}(t) = 9\mathbf{i} - [15(3t+2)^4 - 3]\mathbf{j}$; $\mathbf{r}(t) = -180(3t+2)^3\mathbf{j}$
9 $\mathbf{r}(t) = -3(t-7)^2\mathbf{i} - \frac{t}{\sqrt{t^2-3}}\mathbf{j}$;
$\mathbf{r}(t) = -6(t-7)\mathbf{i} + \frac{3}{(t^2-3)^{\frac{3}{2}}}\mathbf{j}$
10–12 Demonstrations
13 a $-\frac{3}{2}$ b $\frac{16}{3}$ c 1
d $-\frac{\sqrt{3}}{3}$ e e^{-3} (0.0498) f $\frac{\sqrt{3}}{2}$
14 a $\sin(t)$ b $\frac{1}{2}$
15 Proof
16 $t = \frac{2\sqrt{3}}{3}$

5.03

1 D
2 B
3 A
4 C
5 B

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6 a $t^2\mathbf{i}+t\mathbf{j}+\mathbf{c}$

b $t^4\mathbf{i}-\frac{8}{3}t^{\frac{3}{2}}\mathbf{j}+\mathbf{c}$

c $3t^2\mathbf{i}-\frac{2}{5}t^{\frac{5}{2}}\mathbf{j}+\mathbf{c}$

d $(5t+4t^2)\mathbf{i}+(2t+e^{-t})\mathbf{j}+\mathbf{c}$

e $\sin(t)\mathbf{i}-\frac{1}{2}\cos(2t)\mathbf{j}+\mathbf{c}$

f $e^{2t}\mathbf{i}-\frac{3}{2}\sin(2t)\mathbf{j}+\mathbf{c}$

7 a $\mathbf{r}(t)=2e^{2t}\mathbf{i}+(3e^t-3)\mathbf{j}$

b $\mathbf{r}(t)=(t^3+1)\mathbf{i}+(4t^{\frac{3}{2}}+2)\mathbf{j}$

c $\mathbf{r}(t)=[2+\sin(t)]\mathbf{i}+[2-3\cos(t)]\mathbf{j}$

d $\mathbf{r}(t)=(t^2+2)\mathbf{i}-\cos(t)\mathbf{j}$

8 a $\mathbf{r}(t)=t\mathbf{i}-(5t^2-t-1)\mathbf{j}$

b $\mathbf{r}(t)=20t\mathbf{i}-(16t^2-5t)\mathbf{j}$

c $\mathbf{r}(t)=[-4+4\cos(t)]\mathbf{i}+[4+3\sin(t)]\mathbf{j}$

9 a $4\mathbf{i}+\frac{1}{2}\mathbf{j}$ b $\mathbf{0}$

c $\mathbf{i}+\mathbf{j}$ d $-2\mathbf{j}$

e $2\mathbf{i}+(1-e^2)\mathbf{j}$ f $(1-e^{-1})\mathbf{i}+\frac{1}{2}(e^2-1)\mathbf{j}$

10 a $\frac{1}{3}\mathbf{i}+(1-e^{-1})\mathbf{j}$

b $\frac{19}{3}\mathbf{i}+(e^{-2}-e^{-3})\mathbf{j}$

c $21\mathbf{i}+(e^{-1}-e^{-4})\mathbf{j}$

11 Yes

5.04

1 D

2 B

3 E

4 a 13 b 5

c $\sqrt{4t^2+25}$ d $\sqrt{16+49t^2}$

e 3 f $\sqrt{2t+\frac{9}{t^2}}$

5 a $\mathbf{v}=-4\mathbf{j}$, $\mathbf{a}=0$, $|\mathbf{v}|=4$

b $\mathbf{v}=-\frac{a}{t^2}\mathbf{i}+b\mathbf{j}$, $\mathbf{a}=\frac{2a}{t^3}\mathbf{i}$, $|\mathbf{v}|=\sqrt{\frac{a^2}{t^4}+b^2}$

c $\mathbf{v}=-3[\sin(3t)\mathbf{i}-\cos(3t)\mathbf{j}]$,
$\mathbf{a}=-9[\cos(3t)\mathbf{i}+\sin(3t)\mathbf{j}]$, $|\mathbf{v}|=3$

d $\mathbf{v}=6[\cos(2t)\mathbf{i}+\sin(2t)\mathbf{j}]$,
$\mathbf{a}=-12[\sin(2t)\mathbf{i}-\cos(2t)\mathbf{j}]$, $|\mathbf{v}|=6$

e $\mathbf{v}=-3\mathbf{j}+4\mathbf{k}$, $\mathbf{a}=0$, $|\mathbf{v}|=5$

f $\mathbf{v}=5[a\cos(5t)\mathbf{i}-b\sin(5t)\mathbf{j}]$,
$\mathbf{a}=-25[a\sin(5t)\mathbf{i}+b\cos(5t)\mathbf{j}]$,
$|\mathbf{v}|=5\sqrt{a^2\cos^2(5t)+b^2\sin^2(5t)}$

g $\mathbf{v}=2\mathbf{i}+2t\mathbf{j}-4\mathbf{k}$, $\mathbf{a}=2\mathbf{j}$, $|\mathbf{v}|=2\sqrt{5+t^2}$

h $\mathbf{v}=2e^t\mathbf{i}-2e^{-2t}\mathbf{j}$, $\mathbf{a}=2e^t\mathbf{i}+4e^{-2t}\mathbf{j}$, $|\mathbf{v}|=2e^t\sqrt{1+e^{-6t}}$

i $\mathbf{v}=\cos(t)\mathbf{i}+\sin(t)\mathbf{j}+3\mathbf{k}$, $\mathbf{a}=-\sin(t)\mathbf{i}+\cos(t)\mathbf{j}$,
$|\mathbf{v}|=\sqrt{10}$

j $\mathbf{v}=4t\mathbf{i}+2e^{2t}\mathbf{j}-6\sin(2t)\mathbf{k}$, $\mathbf{a}=4[\mathbf{i}+e^{2t}\mathbf{j}-3\cos(2t)\mathbf{j}]$,
$|\mathbf{v}|=2\sqrt{t^2+e^{4t}+9\sin^2(2t)}$

6 a $\mathbf{a}=5\mathbf{i}-2\mathbf{j}$, $\mathbf{r}=(\frac{5}{2}t^2+3)\mathbf{i}-(t^2+4)\mathbf{j}$

b $\mathbf{a}=4\cos(t)\mathbf{i}+3\sin(3t)\mathbf{j}$,
$\mathbf{r}=[4-4\cos(t)]\mathbf{i}-[\frac{1}{3}\sin(3t)-6]\mathbf{j}$

c $\mathbf{a}=-\frac{2}{t^2}\mathbf{i}$, $\mathbf{r}=[2\log_e|t|+5]\mathbf{i}-3t\mathbf{j}$

d $\mathbf{a}=2e^{2t}\mathbf{i}+6t\mathbf{j}$, $\mathbf{r}=(\frac{1}{2}e^{2t}-\frac{1}{2}e^2+2)\mathbf{i}+(t^3+4t-4)\mathbf{j}$

e $\mathbf{a}=6t\mathbf{i}$, $\mathbf{r}=(t^3+9)\mathbf{i}+(4t+7)\mathbf{j}$

7 a $x-2y=0$ b $4x+3y=0$

c $x+2y-9=0$ d $y^2=\frac{16x}{3}$

e $3x-y-11=0$ f $\frac{x^2}{4}+\frac{y^2}{25}=1$

g $y=3x^4-1, x\ge 0$ h $x^2+y^2-4x-8y+19=0$

8 a Proof

b Acceleration is in the opposite direction to displacement.

c Circle centred at the origin, with radius 3 units.

9 a Proof

b Acceleration is perpendicular to velocity.

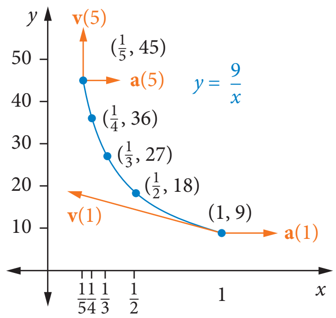
10 a $3t\mathbf{i}+(2t-3)\mathbf{j}$, $(t^2-4)\mathbf{i}+(t+1)\mathbf{j}$

b Proof

c $t=4$, $\mathbf{r}=12\mathbf{i}+5\mathbf{j}$

11 a Rectangular hyperbola

b, d



c $\mathbf{v}(1)=-\mathbf{i}+9\mathbf{j}$, $\mathbf{a}(1)=2\mathbf{i}$, $\mathbf{v}(5)=-\frac{1}{25}\mathbf{i}+9\mathbf{j}$, $\mathbf{a}(5)=\frac{2}{125}\mathbf{i}$

12 a $\mathbf{v}=(3t-2t^2+2)\mathbf{i}$, $\mathbf{r}=\left(\frac{3t^2}{2}-\frac{2t^3}{3}+2t+5\right)\mathbf{i}$

b $t=2$, $\mathbf{r}=9\frac{2}{3}\mathbf{i}$

13 a Proof

b $\mathbf{v}=-4[\sin(t)\mathbf{i}-\cos(t)\mathbf{j}]$, $\mathbf{a}=-4[\cos(t)\mathbf{i}+\sin(t)\mathbf{j}]$

c Velocity is perpendicular to $\mathbf{r}$.

d Acceleration is opposite to $\mathbf{r}$.

14 a Ellipse with axes 6 units and 10 units long with centre at the origin.

b $\mathbf{v}=-3\sin(t)\mathbf{i}+5\cos(t)\mathbf{j}$, $\mathbf{a}=-[3\cos(t)\mathbf{i}+5\sin(t)\mathbf{j}]$

c $t=\frac{n\pi}{2}$ $(n=0, 1, 2, \ldots)$

d The acceleration and position vectors are parallel, equal in magnitude and opposite in direction.

15 a $\mathbf{v} = (t^2 - 4)\mathbf{i} - (3t - 6)\mathbf{j}$
b $t = 2$
c $|\mathbf{a}| = 5$

16 a $0\mathbf{i} + 0\mathbf{j}$
b $2\sqrt{17}$
c $y = 4x - \frac{1}{4}x^2$

17 a $r_P = (2t + 5)\mathbf{i} + (t^2 - 6)\mathbf{j}$, $\mathbf{r}_Q = (3t - 1)\mathbf{i} + 5t\mathbf{j}$
b $t = 6$, $17\mathbf{i} + 30\mathbf{j}$
c $P: y = \frac{1}{4}(x^2 - 10x + 1)$; $Q: 5x - 3y + 5 = 0$

18 a 6 m above the origin
b 4 m s^{-1}
c 0°
d $\mathbf{a} = -10\mathbf{j}$

INVESTIGATION: ROCKETS AND SATELLITES

Some of the physical factors are the strength of the gravitational field, the mass of the rocket/satellite, the type and amount of fuel, the aerodynamics of the rocket and so on.
History should include the German World War 2 rockets, Sputnik and other orbital craft, the Moon landings, space stations and space exploration with unmanned probes and landers.

5.05

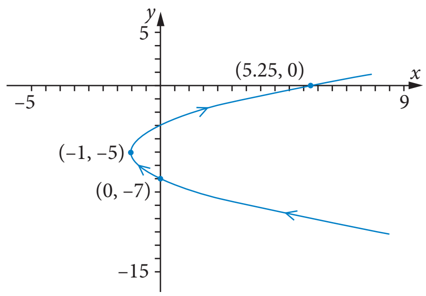
1 C
2 D
3 C
4 C
5 D
6 a 5 m b 2 s c $20\sqrt{3}$ m
7 a 3.61 s b 63.8 m c 15.9 m
d $\mathbf{r}(t) \approx 17.7\,t\mathbf{i} + (17.7t - 4.9t^2)\mathbf{j}$
e 21.8 m
8 a $\mathbf{r}(t) \approx 15\sqrt{3}\,t\mathbf{i} + (15t - 4.9t^2)\mathbf{j}$
b 11.5 m c 3.06 s, 79.5 m
d 28.66 m s^{-1}, $\theta = 25°$ (twice)
9 a 33.1 m b 91.8 m
10 a 39.2 m b 9° or 81°
11 a 24.2 m s^{-1} b 3.5 s
12 14.64 m s^{-1}
13 a 7.6 s b 323.5 m
14 3118 m
15 16.4 m
16 0.62° (0° 37′)
17 31.3 m s^{-1}, 4.5 s
18 Yes, unless diameter > 10 mm
19 83.1 m
20 a Proof b 4.02 m
21 a A 35.1°, B 54.9°
b 2.93 s
c 25.2 m
d $A : B = 0.494 : 1$

5.06

1 E
2 C
3 B
4 A
5 C
6 B
7 D
8 E
9 E
10 a 1.1 rev $\left(\frac{7}{2\pi}\text{rev}\right)$
b 1257 rad (400π rad)
c 50.3 rad s^{-1} (16π rad s^{-1})
11 5 rad s^{-1}
12 a 2.24 rad s^{-1} b 5.39 m s^{-1} c 12.1 m s^{-2}
13 a 2.1 rad s^{-1} b 20.3 rpm
14 23 562 km h^{-1}
15 a 31.4 rad s^{-1} b 0.628 m s^{-1}
16 a 1.8 s b 3.491 rad s^{-1}
c 0.53 m s^{-1} d 0.17 m s^{-1}
e 1.86 m s^{-2}
17 a 4.17 m s^{-1} ≈ 15 km h^{-1} b 11.72 rad s^{-1}
c 0.54 s d 48.8 m s^{-2}
18 a 41.9 rad s^{-1} b 33.5 m s^{-1} c 1404 m s^{-2}
19 a 0.419 rad h^{-1} b 14 661 km h^{-1}
20 a 1.18 rad s^{-1} b 2.65 rad s^{-1} c 3.74 rad s^{-1}
21 62.5 m

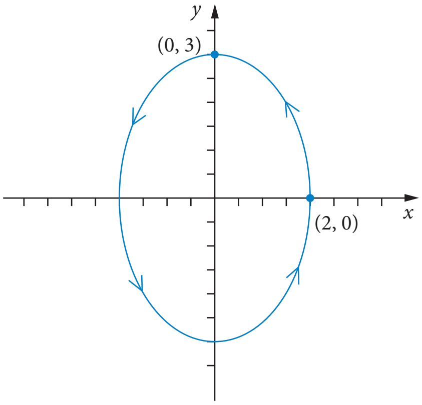
CHAPTER 5 REVIEW

1 B
2 D
3 C
4 E
5 A
6 D
7 C
8 C
9 E
10 C
11 D
12 D
13 a $x = \frac{1}{4}y^2 + \frac{5}{2}y + \frac{21}{4}$
b



14 a $\frac{x^2}{4} + \frac{y^2}{9} = 1$

b Domain = [−2, 2]; Range = [−3, 3]



15 a $6\mathbf{i} - 2\mathbf{j}$

b $\sqrt{51}$

16 a $2e^t\mathbf{i} - 5\cos(5t)\mathbf{j}$

b $\sqrt{4e^{2t} + 25\cos^2(5t)}$

c $2e^t\mathbf{i} + 25\sin(5t)\mathbf{j}$

17 $\mathbf{r}(t) = 15(3t+7)^4\mathbf{i} - 16t\mathbf{j}$; $\mathbf{r}(t) = 180(3t+7)^3\mathbf{i} - 16\mathbf{j}$

18 Demonstration

19 a $\frac{3}{5}\tan(t)$ b $\frac{3\sqrt{3}}{5}$

20 $\mathbf{r}(t) = [\frac{1}{2}\sin(2t)+3]\mathbf{i} + [\cos(t) - 3]\mathbf{j}$

21 $\mathbf{r}(t) = (6t^2 + 10t)\mathbf{i} - [\frac{1}{2}t(5t+30)]\mathbf{j}$

22 $\mathbf{r}(t) = \frac{4}{5}\mathbf{i} + [\frac{1}{2}(e^{-2} - 1)]\mathbf{j}$

23 a i $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$ ii $\mathbf{a} = \mathbf{0}$

iii $|\mathbf{v}| = \sqrt{13}$

b i $\mathbf{v} = 15\cos(3t)\mathbf{i} - 18\sin(3t)\mathbf{j}$

ii $\mathbf{a} = -45\sin(3t)\mathbf{i} - 54\cos(3t)\mathbf{j}$

iii $|\mathbf{v}| = \sqrt{225 + 99\sin^2(3t)}$

24 a $\mathbf{a} = 4\mathbf{i}$, $\mathbf{r} = (2t^2+2)\mathbf{i} + (1-5t)\mathbf{j}$

b $\mathbf{a} = -4\sin(t)\mathbf{i} + 6\cos(t)\mathbf{j}$;

$\mathbf{r} = [4\sin(t) - 5]\mathbf{i} + [6 - 6\cos(t)]\mathbf{j}$

c $\mathbf{a} = 2e^{0.4t}\,\mathbf{i}$; $\mathbf{r} = (12.5e^{0.4t} - 13.65)\mathbf{i} + (4t-3)\mathbf{j}$

25 $x + 2y - 3 = 0$

26 $\mathbf{r} = [V\cos(\theta)]t\,\mathbf{i} + [V\sin(\theta)t - \frac{1}{2}gt^2]\mathbf{j}$,

$\dot{\mathbf{r}} = V\cos(\theta)\mathbf{i} + [V\sin(\theta) - gt]\mathbf{j}$,

$\ddot{\mathbf{r}} = -g\mathbf{j}$ or $x = [V\cos(\theta)]t$, $y = [V\sin(\theta)]t - \frac{1}{2}gt^2$,

$\dot{x} = V\cos(\theta)$, $\dot{y} = V\sin(\theta) - gt$,

$\ddot{x} = 0$, $\ddot{y} = -g$

27 a 58.8 m b 6.93 s c 135.8 m

d 32.05 m s^{-1} at ± 52.3°

28 a $\mathbf{r}(t) = 20t\mathbf{i} + (50 + 25t - 4.9t^2)\mathbf{j}$

b About 6.64 s. c About 132.8 m.

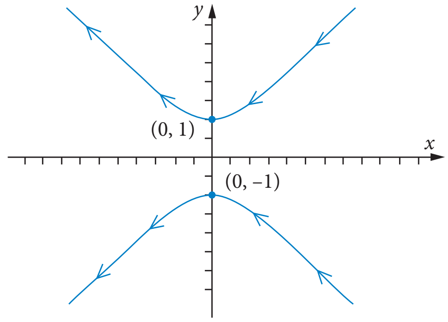
d About 81.8 m. e About 141.9 m.

29 $a = \omega^2 r$ and $a = \frac{v^2}{r}$

30 a $\frac{40\pi}{3}$ rad s^{-1} b 28π m s^{-1} (about 88 m s^{-1})

31 $\omega = \frac{2}{3}$ rad s^{-1}, $a = 5\frac{1}{3}$ m s^{-2}

32 $y^2 - x^2 = 1$



33 $t = \sqrt{3}$ s

34 a (0, 0) b 8.24 m s^{-1}

c $y = 4x - \frac{x^2}{4}$

35 a $t = \sqrt{\frac{2h}{g}}$ b $V_t = \sqrt{V^2 + 2hg}$

c $y = h\left(1 - \frac{x^2}{d^2}\right)$

36 $\mathbf{v} = 2\mathbf{i} + (4t - \frac{1}{2}t^2)\mathbf{j}$, $\mathbf{r} = (2t+5)\mathbf{i} + \left(2t^2 - \frac{t^3}{6}\right)\mathbf{j}$

37 a 21 m s^{-1} b 3.03 s

38 a 4.28 s b 70.4 m c 31.9 m

39 No, it will just hit the top.

40 $\omega = 5.24$ rad s^{-1}, $v = 0.942$ m s^{-1}, $a = 4.93$ m s^{-2}

41 $v \approx 29.89$ km s^{-1}, $a \approx 0.005\,95$ m s^{-2}

INVESTIGATION: ROOTS OF UNITY

The cube roots of unity are $\cos\left(\frac{2k\pi}{3}\right) + i\sin\left(\frac{2k\pi}{3}\right)$ for $k = 0, 1, 2$.

Choose, say, $z_3 = \text{cis}\left(\frac{2\pi}{3}\right)$.

Choose, say, $z_8 = \text{cis}\left(\frac{6\pi}{8}\right) = \text{cis}\left(\frac{3\pi}{4}\right)$

Then $z_3 \times z_8 = \text{cis}\left(\frac{2\pi}{3} + \frac{3\pi}{4}\right) = \text{cis}\left(\frac{17\pi}{12}\right)$

and $z_3 \div z_8 = \text{cis}\left(\frac{2\pi}{3} - \frac{3\pi}{4}\right) = \text{cis}\left(-\frac{\pi}{12}\right)$

Now $(z_3 \times z_8)^{24} = \text{cis}\left(\frac{17\pi}{12} \times 24\right) = \text{cis}(17 \times 2\pi) = 1$

and $(z_3 \div z_8)^{24} = \text{cis}\left(-\frac{\pi}{12} \times 24\right) = \text{cis}(-2\pi) = 1$

Thus $z_3 \times z_8$ and $z_3 \div z_8$ are both roots of unity.

Any product or sum of nth and mth roots of unity can be proven to be a root of unity by raising it to the power that is twice the lowest common multiple of n and m.

6.01

1 a $z = \cos\left(\frac{-2\pi}{3}\right) + i\sin\left(\frac{-2\pi}{3}\right)$ or $z = 1$ or

$z = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

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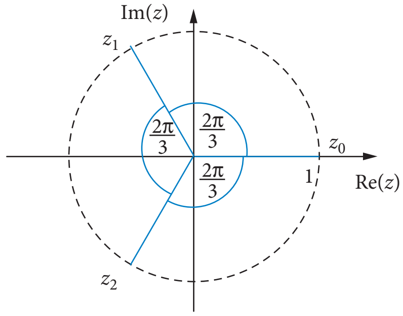
b

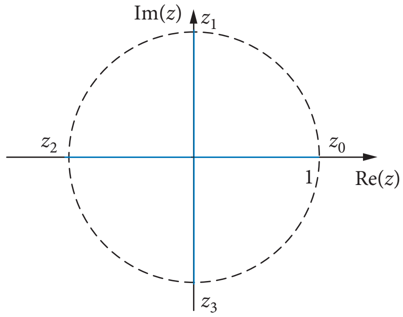


The roots are equally spaced on the unit circle, separated by angles of $\frac{2\pi}{3}$, with one root being 1.

c $z=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ or $z=1$ or $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

2 a $z=-1$ or $z=i$ or $z=1$ or $z=-i$

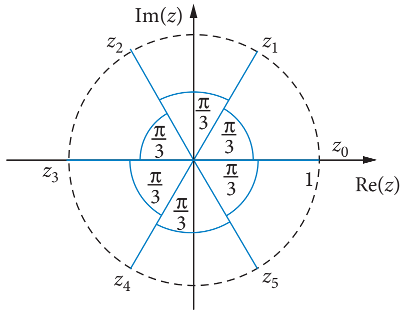
b



The roots are equally spaced on the unit circle separated by angles of $\frac{\pi}{2}$, with one root being 1 and another being −1. They are the powers of i.

3 a $z=\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)$ or $z=\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)$ or $z=1$ or $z=\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)$ or $z=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)$ or $z=-1$

b



The roots are equally spaced on the unit circle separated by angles of $\frac{\pi}{3}$, with one root being 1 and another being −1.

4–6 Demonstrations

7 a $z=-1$ or $z=i$ or $z=1$ or $z=-i$

b $z=\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)$ or $z=\cos(0)+i\sin(0)$ or $z=\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)$ or $z=\cos(\pi)+i\sin(\pi)$

8 a $z=i$ or $z=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ or $z=-i$ or $z=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ or $z=1$ or $z=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ or $z=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ or $z=-1$

b $z=\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)$ or $z=\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)$ or $z=\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)$ or $z=\cos(0)+i\sin(0)$ or $z=\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)$ or $z=\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)$ or $z=\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$ or $z=\cos(\pi)+i\sin(\pi)$

9 a $z=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ or $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$ or $z=1$ or $z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ or $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ or $z=-1$

b $z=\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)$ or $z=\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)$ or $z=\cos(0)+i\sin(0)$ or $z=\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)$ or $z=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)$ or $z=\cos(\pi)+i\sin(\pi)$

10–14 Proofs

6.02

1 $\cos\left[\frac{(4k-1)\pi}{16}\right]+i\sin\left[\frac{(4k-1)\pi}{16}\right]$ for $k=-3,-2,\ldots,3,4$

2 $\cos\left(\frac{-13\pi}{18}\right)+i\sin\left(\frac{-13\pi}{18}\right)$, $\cos\left(\frac{-\pi}{18}\right)+i\sin\left(\frac{-\pi}{18}\right)$ or $\cos\left(\frac{11\pi}{18}\right)+i\sin\left(\frac{11\pi}{18}\right)$

3 $\cos\left[\frac{(8k+3)\pi}{16}\right]+i\sin\left[\frac{(8k+3)\pi}{16}\right]$ for $k=-2,-1,0,1$

4 a $-4i$, $4\left[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right]$ or $4\left[\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right]$

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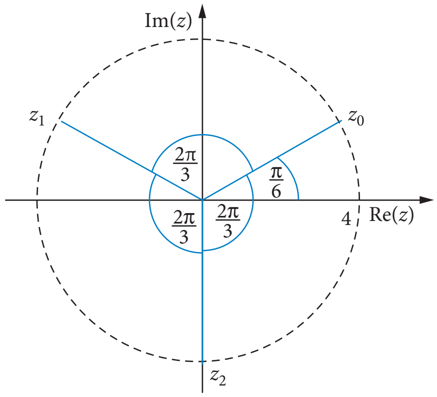
b

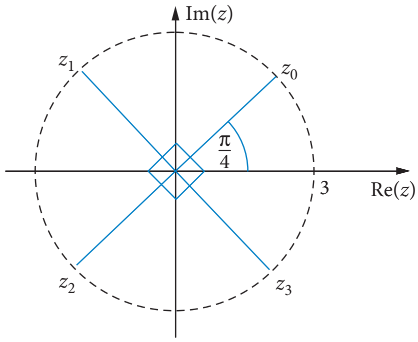


The roots are equally spaced on a circle of radius 4, centred at the origin. One of the roots is $-4i$.

5 a $3\left\{\cos\left[\frac{(2k+1)\pi}{4}\right]+i\sin\left[\frac{(2k+1)\pi}{4}\right]\right\}$ for

$k = -2, -1, 0, 1$

b



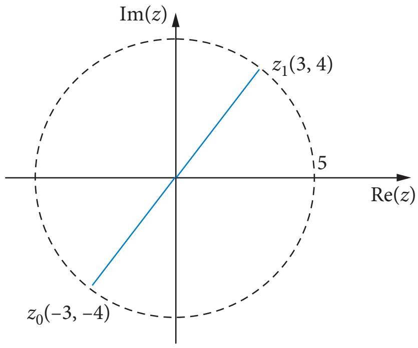
The roots are equally spaced on a circle of radius 3, centred at the origin.

6 a $5[\cos(0.9273) + i\sin(0.9273)]$ or

$5[\cos(-2.2143) + i\sin(-2.2143)]$

b $-3 - 4i$ or $3 + 4i$

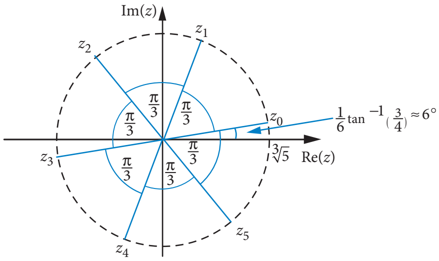
c



7 a $\sqrt[3]{5}\cos\left[\frac{1}{6}\left(\tan^{-1}\left(\frac{3}{4}\right)+2k\pi\right)\right]+$

$i\sin\left[\frac{1}{6}\left(\tan^{-1}\left(\frac{3}{4}\right)+2k\pi\right)\right]$ for $k = -3, -2, \ldots, 2$

b

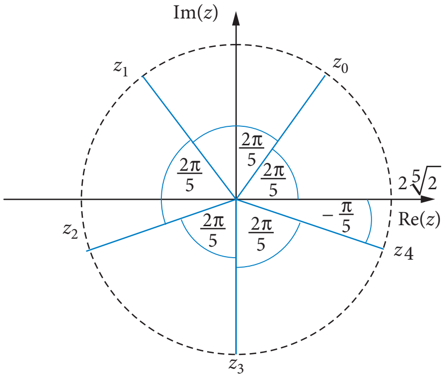


The roots are equally spaced on a circle of radius $\sqrt[3]{5} \approx 1.71$, centred at the origin.

8 a $2\sqrt[5]{2}\left\{\cos\left[\frac{(4k-1)\pi}{10}\right]+i\sin\left[\frac{(4k-1)\pi}{10}\right]\right\}$ for

$k = -2, -1, 0, 1, 2$

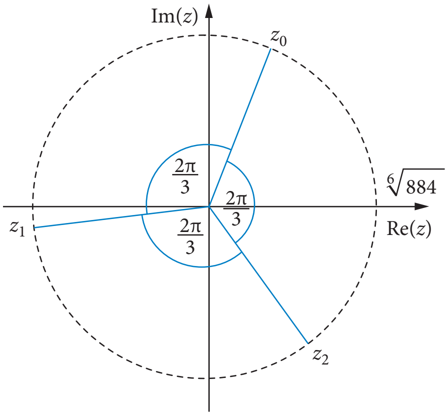
b



The roots are equally spaced on a circle of radius $2\sqrt[5]{2} \approx 2.30$, centred at the origin.

9 a $-3.078 - 0.353i$, $1.233 + 2.842i$ or $1.845 - 2.489i$

b



The roots are equally spaced on a circle of radius $\sqrt[6]{884} \approx 3.10$, centred at the origin.

10 Proof

6.03

1 C

2 E

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3 a $18 + 3i$ b $-10 - 17i$
c $-87 - 12i$ d $93i$
e $-150 - 15i$

4 a $-610 + 1869i$ b $-6 + 103i$
c $2939 - 957i$ d $970 + 263i$
e $-70 + 39i$

5 a $44 + 2i$ b $-96 + 13i$
c $-16 + 137i$ d $107 + 16i$
e $104 + 27i$

6 a $1312 + 715i$ b $3025 + 1905i$
c $-5383 - 8015i$ d $-2131 + 4351i$
e $3013 + 7373i$

7 a $-28 + 101i$ b $-646 - 880i$
c $842 - 1129i$ d $-1372 + 647i$
e $-484 + 138i$

8 a $-2827 + 484i$ b $-12\,866 + 6397i$
c $892 + 2299i$ d $-153\,935 - 15\,922i$
e $-151 - 160i$

9 Not true, disprove by a counterexample such as $p(z) = z^2$, $a = b = 1$. It is true for polynomials of degree 0 or 1, but not those of higher degree.

6.04

1 a $(-3 - 2i)z^3 + (9 + 7i)z^2 + (4 - 4i)z + 12 - 3i$
b $(-3 - 2i)z^3 + (1 - 3i)z^2 - 2z + 4 + 9i$
c $(-3 - 2i)z^3 + (7 + 4i)z^2 + (-1 - 6i)z + 11 + 5i$
d $(-2 - 3i)z^2 + (-5 - 2i)z - 1 + 8i$
e $(-2 + 18i)z^4 + (22 - 24i)z^3 + (8 + 11i)z^2 + (-19 - 4i)z + 24 - 10i$
f $(-2 - 23i)z^5 + (-3 + 33i)z^4 + (9 + 3i)z^3 + (48 + 22i)z^2 + (22 - 21i)z + 50 - 36i$
g $(-2 - 10i)z^5 + (4 + 30i)z^4 + (-1 - 38i)z^3 + (11 + 38i)z^2 + (3 - 42i)z + 18 + 25i$

2 a $(4 + i)z^2 + (15 + 3i)z + 44 + 6i + \dfrac{127 + 20i}{z - 3}$
b $(4 + i)z^2 + (1 + 8i)z - 17 - i + \dfrac{-3 - 32i}{z - 2i}$
c $(4 + i)z^2 + (-1 - i)z - 2i + \dfrac{-5 + 4i}{z + 1}$
d $(4 + i)z^2 + (12 - 2i)z + 21 - 19i + \dfrac{18 - 57i}{z - 2 + i}$
e $(4 + i)z^2 + (-13 + 13i)z - 14 - 94i + \dfrac{413 + 228i}{z + 3 - 4i}$

3 a $-4iz^3 + (-3 + 2i)z^2 + (4 + 6i)z + 9 - 8i + \dfrac{-9 - 18i}{z + 2i}$
b $-4iz^3 + (9 - 10i)z^2 + (37 - 21i)z + 129 - 26i + \dfrac{420 + 51i}{z - 3 - i}$
c $-4iz^3 + (-3 + 10i)z^2 + (26 - 14i)z - 83 - 24i + \dfrac{125 + 214i}{z + 2 + 2i}$
d $-4iz^3 + (13 + 18i)z^2 + (-88 - 46i)z + 441 + 8i + \dfrac{-1773 + 850i}{z + 4 - 2i}$
e $-4iz^3 + (5 - 18i)z^2 + (25 - 90i)z + 122 - 450i + \dfrac{617 - 2250i}{z - 5}$

4 a $(4 + 2i)z^5 + (5 - 3i)z^4 + (4 - 2i)z^3 + (5 + 3i)z^2 + (-7 - 1i)z - 2 - 2i$
b $(-4 - 2i)z^5 + (5 - 3i)z^4 + (-10 + 8i)z^3 + (-1 - 11i)z^2 + (-7 - i)z - 8 + 8i$
c $(5 - 3i)z^4 + (6 + 4i)z^3 + (2 - 4i)z^2 + (-10 + i)z - 2 - 2i$
d $(-4 - 2i)z^5 + (2 + 6i)z^3 + (-3 - 7i)z^2 + (-3 + 2i)z$
e $(34 + 22i)z^8 + (52 - 36i)z^6 + (42 + 52i)z^5 + (11 + 29i)z^4 + (5 - 107i)z^3 + (44 + 6i)z^2 + (1 + 21i)z - 16 - 30i$
f $(26 - 2i)z^9 + (-18 + 6i)z^8 + (36 - 58i)z^7 + (4 + 44i)z^6 + (-62 - 48i)z^5 + (-20 - 4i)z^4 + (-28 + 18i)z^3 + (-50 - 48i)z^2 + (-26 + 32i)z + 34i$
g $(48 - 22i)z^7 + (-30 + 24i)z^6 + (13 - 15i)z^5 + (-59 - 65i)z^4 + (-40 + 62i)z^3 + (9 - 33i)z^2 + (-17 + 13i)z + 34i$

5 a $-252 - 371i$ b $54 - 13i$
c $1117 + 1553i$ d $-10 - 21i$
e $269 - 143i$

6 a $83 - 239i$ b $3566 - 2418i$
c $342 + 13i$ d $-4249 - 2818i$
e $603 + 5181i$

7 a $-6648 - 16\,352i$ b $-964 + 58i$
c $-24\,028 + 12\,358i$ d $82\,482 - 10\,442i$
e $-44 - 30i$

8–15 Proofs using the factor theorem.

6.05

1 E
2 B
3 C
4 $z = 2$, $z = i$, or $z = -i$
5 $z = -3$, $z = 1 - 2i$, or $z = 1 + 2i$
6 $z = 1$, $z = -1$, $z = 2 - i$, or $z = 2 + i$
7 $z = -1$, $z = 2$, $z = 3 - i$, or $z = 3 + i$
8 $z = -1$, $z = 2$, $z = -3$, $z = 2 - 3i$, or $z = 2 + 3i$
9 $z = 2$, $z = 3$, $z = -3$, $z = -1 + 2i$, or $z = -1 - 2i$
10 $(z + 3)(z + 1 + i)(z + 1 - i)$
11 $(z - 1)(z - 2)(z - 2 + 2i)(z - 2 - 2i)$
12 $(z + 1)(z - 2)(z + 2)(z - 1 + 3i)(z - 1 - 3i)$

6.06

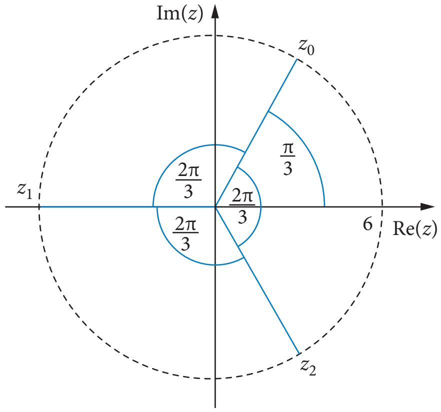
1 A
2 C
3 $z = 1$, $z = -2$ and $z = 1 + 2i$
4 $z = -2$, $z = 3$ and $z = 2 - 3i$
5 $z = -i$, $z = 3i$ and $z = 2 + i$
6 $z = -i$, $z = -2i$ and $z = -3 - 2i$
7 $z = 1$, $z = -2$ and $z = 3 + i$ (twice)
8 $z = 2$, $z = -2$, $z = \frac{1}{5}i$ and $-\frac{3}{2}i$
9 $z = i$, $z = 3i$, $z = \frac{3}{4}i$ and $z = -\frac{4}{3}i$
10 $z = 2i$, $z = -3i$, $z = 3 + 2i$ and $z = -2 + 3i$

CHAPTER 6 REVIEW

1 C
2 B
3 D
4 C
5 A
6 C
7 A
8 E
9 B
10 E

11 a $6\left[\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right]$, -6, and $6\left[\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right]$

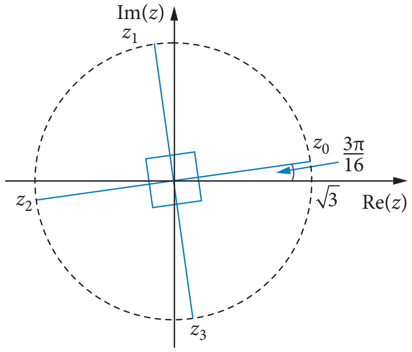
b



12 a $\sqrt{3}\left[\cos\left(\frac{3\pi+8k\pi}{16}\right)+i\sin\left(\frac{3\pi+8k\pi}{16}\right)\right]$

for $k=-2, -1, 0, 1$

b



13 $-4+2i$, $-5-5i$, $-16-2i$
14 $92-40i$, $6-3i$, $54-464i$
15 a $(1-2i)z^3+iz+3+2i$
b $(-6-3i)z^5+(15-7i)z^4+18iz^3+(-10+5i)z^2+(-14+8i)z+6i$
c $(1-2i)z^2-z-5+i+\frac{-4+8i}{z-1+i}$
16 a $1+7i$ b $-122+108i$ c $36-38i$
17 Use $p(z+1-2i)=0$
18 Proof
19 Proof
20 $z=3$, $z=-\frac{1}{2}-\frac{\sqrt{11}}{2}i$ or $z=-\frac{1}{2}+\frac{\sqrt{11}}{2}i$
21 $z=-1$, $z=2$ or $z=-3i$ (twice)
22 $z=2i$, $z=-2i$, $z=\frac{2}{3}i$ and $z=-\frac{3}{2}i$

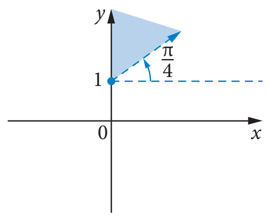
MIXED REVISION 2

Multiple choice

1 B
2 E
3 E
4 D
5 D
6 D
7 A
8 E
9 E

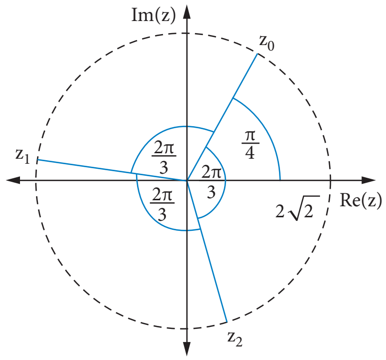
Short answer

1



2 a $2\sqrt{2}\left[\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right]$,
$2\sqrt{2}\left[\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)\right]$,
$2\sqrt{2}\left[\cos\left(-\frac{5\pi}{12}\right)+i\sin\left(-\frac{5\pi}{12}\right)\right]$

b



3 a $\mathbf{a}=3\mathbf{i}$, $\mathbf{r}=(\frac{3}{2}t^2+2)\mathbf{i}+(5t-3)\mathbf{j}$
b $\mathbf{a}=-3\sin(t)\mathbf{i}+2\cos(2t)\mathbf{j}$,
$\mathbf{r}=[3\sin(t)-5]\mathbf{i}+\frac{1}{2}[1-\cos(2t)]\mathbf{j}$
4 a $w_3=-i(w_1-w_2)+w_2$
b $w_4=(w_3-w_1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+w_1$
5 31 416 km h^{-1}

6 $2iz^2 + (-2 - 4i)z + 6 + 9i + \frac{-13 - 13i}{z + 2 - i}$

Application

1 Circle is $x^2 + \left(y + \frac{3}{4}\right)^2 = \frac{1}{16}$

2 arg $(w + 2) = \theta$, $PO = BO$ so $\angle BPO = \theta$ (isosceles triangle) then $\angle POA = 2\theta$ (exterior angle of triangle PBO. $\therefore$ arg $(w) = 2\theta$.

3 $z = 2, z = -3, -\frac{1}{3} - \frac{\sqrt{5}}{3}i, -\frac{1}{3} + \frac{\sqrt{5}}{3}i$

4 $z = i, z = -\frac{4}{5}i, z = \frac{5}{4}i$

5 a Proof

b Acceleration is in the opposite direction of displacement.

c Circle centred at the origin with a radius of 4 units

6 a 29.5 m

b They do not collide, the second ball passes over the top of the first.

7.01

1 D
2 E
3 E
4 C
5 A
6 C
7 A
8–12 Proofs

7.02

1 C
2 A
3 D
4 B
5 D
6 A
7 312
8 $\frac{1}{3}$
9, 10 Proofs

INVESTIGATION: DEFINITE INTEGRALS OF $\frac{1}{x}$

For $x > 0$ and for $x < 0$ the graph is anti-symmetric (it is an odd function).
The physical areas between the curve and the x-axis either side of the y-axis are the same.

$$\int_{-3}^{-1} \frac{1}{x} dx = -\int_{1}^{3} \frac{1}{x} dx = \int_{3}^{1} \frac{1}{x} dx$$

7.03

1 a $\frac{1}{3}\log_e|3x + 2| + c$

b $\frac{1}{9}\log_e|9x - 7| + c$

c $-\frac{1}{3}\log_e|4 - 3x| + c$

d $\frac{5}{2}\log_e|5 - 2x| + c$

2 C
3 B
4 B
5 E
6 A

7 a $\frac{1}{3}\log_e\left|3x^2 + 2\right| + c$

b $2\log_e|\tan(x)| + c$

c $\frac{1}{2}\log_e\left|e^{2x} + 1\right| + c$

d $2\log_e\left|x^3 + 2x\right| + c$

8–10 Proofs

7.04

1 a $\frac{3}{\sqrt{1 - 9x^2}}$ b $\frac{3x^2}{1 + \left(x^3 - 2\right)^2}$

c $\frac{-2x}{\sqrt{1 - x^4}}$ d $\frac{8x^3 + 1}{1 + \left(2x^4 + x\right)^2}$

2 E
3 D
4 A
5 A
6 E

7 a $\frac{1}{2}\sin^{-1}(2x) + c$

b $-\frac{4}{3}\tan^{-1}(3x) + c$

c $\frac{\pi}{4}$ d $\frac{\pi}{3}$

8 a $\arcsin\left(\frac{x}{6}\right) + c$

b $\arccos\left(\frac{x}{10}\right) + c$

c $\frac{1}{7}\arctan\left(\frac{x}{7}\right) + c$

d $\arcsin\left(\frac{x}{2\sqrt{3}}\right) + c$

e $\frac{1}{\sqrt{6}}\arctan\left(\frac{x}{\sqrt{6}}\right) + c$

9 $\frac{1}{2}$[arctan (3) + arctan (1.5)]

10 1.1007

11 $\dfrac{6}{x^2+2}=3\sqrt{2}\left(\dfrac{\frac{1}{\sqrt{2}}}{\left(\frac{x}{\sqrt{2}}\right)^2+1}\right)$ where $u(x)=\dfrac{x}{\sqrt{2}}$.

Hence $\int\dfrac{6}{x^2+2}dx=3\sqrt{2}\arctan\left(\dfrac{x}{\sqrt{2}}\right)+c$.

12–14 Proofs

7.05

1 a $\dfrac{1}{x-2}-\dfrac{1}{x-1}$

b $\dfrac{3}{4(x+2)}+\dfrac{9}{4(x-6)}$

c $\dfrac{3}{7(2x+3)}+\dfrac{2}{7(x-2)}$

d $\dfrac{9}{49(2x+3)}+\dfrac{20}{49(x-2)}+\dfrac{4}{7(x-2)^2}$

2 a $\log_e\left|\dfrac{x-2}{x-1}\right|+c$ b $\dfrac{3}{4}\log_e\left(\dfrac{256}{375}\right)$

c $\dfrac{1}{7}\log_e\left|\dfrac{x-2}{2x+3}\right|+c$ d $\dfrac{3}{49}\log_e\left(\dfrac{3}{10}\right)+\dfrac{1}{7}$

3 B

4 A

5 C

6 B

7 $x+\log_e\left|\dfrac{(x-2)^4}{x-1}\right|+c$

8 $\dfrac{x^2}{(x^2-1)}=1+\dfrac{1}{(x^2-1)}, x+\dfrac{1}{2}\log_e\left|\dfrac{x-1}{x+1}\right|+c$

9 $\dfrac{1}{2}-\dfrac{1}{2}\log_e(3)$

10 $-\log_e(2)$

7.06

1 B

2 C

3 C

4 B

5 E

6 a $\dfrac{1}{4}e^{2x}(2x-1)+c$

b $\dfrac{1}{4}[2x\sin(2x)+\cos(2x)]+c$

c $\dfrac{1}{4}x^2[2\log_e(2x)-1]+c$

d $\dfrac{1}{4}(e^2+1)$ e $\dfrac{\pi}{4}$

7 a $\dfrac{1}{3}[\log_e(3)-2]$

b $x\sin^{-1}(x)+\sqrt{1-x^2}+c$

c $\dfrac{1}{4}x^2[2\log_e(x)-1]+c$

d $\dfrac{\pi}{6}-\dfrac{\sqrt{3}}{2}$

e $x\cos^{-1}(3x)-\dfrac{1}{3}\sqrt{1-9x^2}+c$

8 $-x^3\cos(x)+3x^2\sin(x)+6x\cos(x)-6\sin(x)+c$

9 $0.25e^2-0.75$

CHAPTER 7 REVIEW

1 D

2 E

3 D

4 B

5 A

6 C

7 E

8 B

9 A

10 D

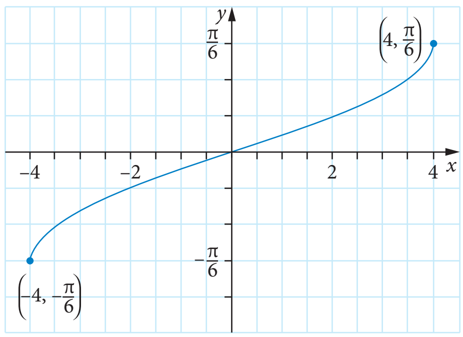
11 $\dfrac{1}{3}$

12 a $-\dfrac{1}{3}\log_e|2-3x|+c$ b $-\dfrac{2}{7}\log_e|7x+2|+c$

13 a $\dfrac{1}{6}\log_e|3x^2+3|+c$ b $4\log_e\left|e^{\frac{x}{2}}+1\right|+c$

14 a $\dfrac{-1}{\sqrt{16-x^2}}$ b $\dfrac{2x}{\sqrt{\frac{3}{4}-x^4+x^2}}$

15



$y=\dfrac{1}{3}\sin^{-1}\left(\dfrac{x}{4}\right)$

Domain: $x\in[-4, 4]$

16 a $\dfrac{1}{9(x-6)}-\dfrac{1}{9(x+3)}$

b $\dfrac{13}{49(x+1)}+\dfrac{36}{49(x-6)}-\dfrac{1}{7(x+1)^2}$

17 a $-\dfrac{3}{4}\log_e\left(\dfrac{375}{256}\right)$

b $\dfrac{1}{63}\log_e\left|\dfrac{(3x-4)^{16}}{(x+1)^9}\right|+\dfrac{1}{3}x+c$

18 $\dfrac{1}{4}e^{2x}(2x^2-2x+1)$

19 $\dfrac{1}{2}[\log_e(2)-1]$

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20 Proof

21 Proof

22 $\sin^{-1}\left(\frac{2x}{5}\right)=\sin^{-1}\left(\frac{x}{\frac{5}{2}}\right)$

$\therefore \int \frac{1}{\sqrt{\frac{25}{4}-x^2}}dx=\sin^{-1}\left(\frac{2x}{5}\right)+c.$

23 $\frac{1}{2}\left[2x^2\sin(2x)+2x\cos(2x)-\sin(2x)\right]+c.$

8.01

1 a $\frac{dy}{dx}=\frac{-6x+5y}{-5x+3y^2}$ b $\frac{dy}{dx}=\frac{-y}{x}$

c $\frac{dy}{dx}=-\frac{2y^2}{3x^2}$ d $\frac{dy}{dx}=\frac{-2y}{x+2\sqrt{y}-2y^{\frac{3}{2}}}$

2 a $\frac{dy}{dx}=-x$ b $y=-\frac{1}{2}x^2+\frac{5}{2},\frac{dy}{dx}=-x$

3 a $\frac{dy}{dx}=\frac{1}{2y+2}$

b i $x=-3+2y+y^2$ ii $\frac{dy}{dx}=\frac{1}{2y+2}$

4 a $\frac{dy}{dx}=\frac{\cos(x+y)\cos^2(y)}{1-\cos(x+y)\cos^2(y)}$

b $\frac{dy}{dx}=\frac{2y-e^{2y}}{2xe^{2y}-2x}$

c $\frac{dy}{dx}=5x+5y-1$

d $\frac{dy}{dx}=\frac{2x\cos(y)}{e^y+x^2\sin(y)}$

5 a $-\frac{5}{3}$ b $3x-5y-30=0$

6 $10x-9y-20=0$

7 $2x+9y=20,\ 2x+9y=-20$

8 $25x-15y=132$

9 $9x-8y=26,\ 9x-8y=-26$

10 $\left(0,\ \frac{100}{9}\right)$

11–14 Proofs

8.02

1 a $8x^3$ b $10e^{2x}$

2 a 297 b −6084 c $3\log_e(4)+3$

3 $3\sqrt{2}$

4 8100 $mm^3\,s^{-1}$

5 91.2π $mm^3\,s^{-1}$

6 $\frac{1638\pi}{25}$ $cm^2\,s^{-1}$

7 $\frac{8901\pi}{175}$ $cm^3\,s^{-1}$

8 40 units per second

9 −0.52 ms^{-1}, i.e. moving down at the rate of 0.52 ms^{-1}

10 2411.472 cms^{-2}

11 $\frac{15}{7}$ houses per year

12 a Proof b $\frac{2}{3}$ cms^{-1}

13 0.57 mm^3s^{-1}

14 $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}=\frac{1}{4\pi r^2}\times -k(4\pi r^2)=-k$, radius will decrease at a constant rate of k ($k>0$)

15 $\frac{2}{3\pi}$ cms^{-1}

16 a Proof b $\frac{dD}{d\alpha}=-\frac{5}{\sin^2(\alpha)}$

c −39 (decreasing by 39 radians/hour)

d No, the angle will reach zero after a few seconds.

8.03

1 a $y=x^3-2x^2+7x+c$

b $y=\frac{8}{3}\sin(3t)+c$

c $y=6w^4-12\log_e|w|+c$

d $y=-2e^{-3x}-4e^{2x}+c$

e $y=\frac{(5+2x)^5}{10}+c$

f $y=\frac{1}{4(3-2x)^2}+c$

g $x=\tan^{-1}\left(\frac{y}{4}\right)+c$

h $x=-\sin^{-1}\left(\frac{y}{2}\right)+c$

2 a $y=2\log_e|x(x-5)|-2\log_e(6)$

b $y=3\log_e|x|-\frac{1}{x}+4$

c $y=\frac{6}{5}\log_e\left(\frac{6|x-2|}{|x+3|}\right)+1$

d $y=\frac{3\sqrt{2}}{2}\tan^{-1}\left(\frac{\sqrt{2}(x+2)}{2}\right)-\frac{3\sqrt{2}}{2}\tan^{-1}\left(\sqrt{2}\right)+2$

e $y=\tan(x)-x+\frac{\pi}{4}$

f $y=-\frac{4}{3}\cos^2(3x)+2$ or $-\frac{2}{3}\cos(6x)+\frac{4}{3}$

g $x=\frac{1}{3}\sin^{-1}\left(\frac{3y}{2}\right)+1$

h $x=0.3\arctan(0.4y)+1$

i $y=0.2(5x^2-2)^5-0.6$

j $y=\cos^3(x)-\frac{1}{8}$

3 a $x+\frac{1}{x-1}$

b $y=-\log_e|x-1|-\frac{x^2}{2}+c$

4 a $\log_e(x)+1$

b $y=x\log_e(x)-x+c$

5 $h=-5t^2+20t+1.5$, 20.25 m

6 $P(n)=100\log_e(n^2+40n+1)$, \$1704

7 a $h=25t-0.8t^2$ b 87.2 m c Proof

8 $Q=100x^2$, 1.44 J

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9 $C = \frac{2n^3}{3000} - \frac{n^2}{2} + 250n + 20000$ dollars

10 Momentum $= 8t - t^2 + 50$ N s

INVESTIGATION: MERCURY POLLUTION

Original amount of mercury = 200 in 1.7 million ≈ 117.6 ppm

$\frac{dA}{dt} = -\frac{20}{1700}A \approx -0.01176A$ ppm/h

$A = 117.6e^{-0.01176t}$ ppm after t hours.

'Safe' level in fish varies between countries but is about 1 ppm. 'Safe' level in drinking water is much less; about 0.002 ppm.

If the lake is used as a source of drinking water, it would take about 40 days to reduce to a safe level, so it could not be used for this period.

Pollution of the continental shelf leads to higher levels of mercury in seafood.

8.04

1 a $y = \pm Ae^{3x}$

b $y = \pm Ae^{2x} + \frac{1}{2}$

c $y = \frac{1}{3}\log_e\left[\frac{-1}{3(x+c)}\right],\ x + c < 0$

d $y = \tan(x+c),\ -\frac{\pi}{2} - c < x < \frac{\pi}{2} - c$

e $y = 4\sin(x+c), -\frac{\pi}{2} - c \le x \le \frac{\pi}{2} - c$

f $y = \frac{-2e^{2x}}{e^{2x} - c}$

2 a $y = 2e^x - 1$

b $y = 4e^{x-2}$

c $y = \frac{e^4 - 25e^{4x}}{5e^{4x} - e^4},\ \frac{1}{5e^{4x} - e^4} > 0$

d $y = \sin(x - \pi),\ \frac{\pi}{2} \le x \le \frac{3\pi}{2}$

3 $Q \approx 100e^{-0.000\,121t}$ % (t in years), about 13 300 years

4 $A \approx 1.5e^{0.0817t}$ m^2 (t in days), 183 or 184 days

5 $N \approx 20e^{0.752t}$ (t in hours), 5.2 h

6 $L \approx 52.43e^{0.223t}$ times the acceptable level (t in hours). At t_0, $L \approx 52.43$ times the acceptable level, the sample was unacceptable.

7 $\frac{dT}{dt} = \frac{3750 - 7.5T}{6000}$, 170°C, 500°C

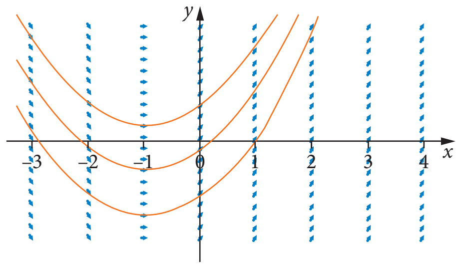
8 $T = 25 + 75e^{-0.155t}$ °C (t in min), another 5 min 5 s

9 $T = 1100 - 1080e^{-0.3646t}$ °C (t in min), 12:31 p.m., 1100 °C

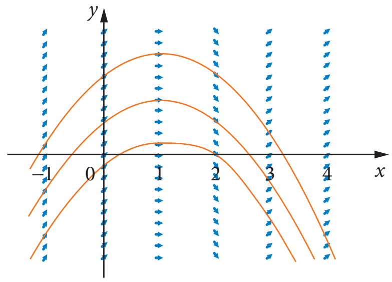
10 $v = \frac{gm}{k}\left(1 - e^{-\frac{k}{m}t}\right)$. It reaches a final velocity of $\frac{gm}{k}$.

INVESTIGATION: GRADIENT FIELDS

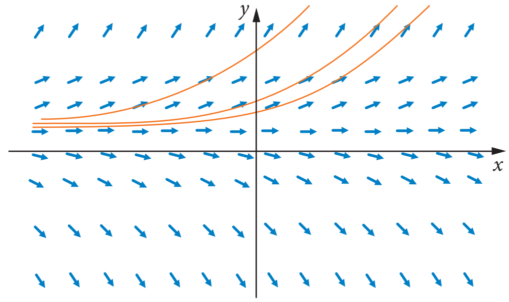
$\frac{dy}{dx} = 0.5 + 0.5x$ is shifted to the left and flatter than $\frac{dy}{dx} = 0.8x$.



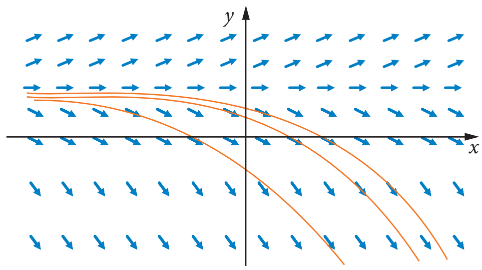
$\frac{dy}{dx} = 0.6 - 0.6x$ is shifted to the left, flatter and upside down compared to $\frac{dy}{dx} = 0.8x$.



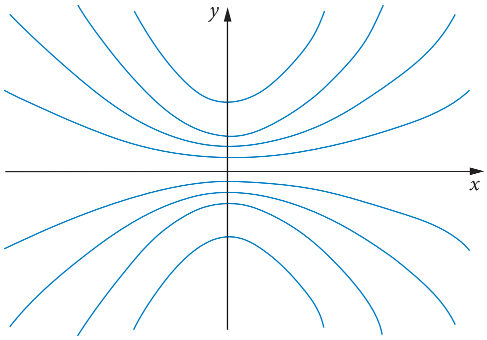
$\frac{dy}{dx} = y + 0.5$ is shifted up and steeper compared to $\frac{dy}{dx} = 0.5y$.



$\frac{dy}{dx} = 1 - 0.6y$ is upside down, shifted up and a little steeper compared to $\frac{dy}{dx} = 0.5y$.



$\frac{dy}{dx} = xy$ is somewhat like the following.



8.05

1 $y = Ae^{0.5x^2 - 3x}$

2 $g = Ae^{6x^2+4x} + \frac{7}{4}$

3 $P = \frac{-2}{V^2 - 3}$

4 $v = \frac{3}{4}e^{6t^2+20t} + \frac{1}{4}$

5 $y = Ae^{0.5x^2} - 1$

6 $f = \frac{6e^{t^2}}{10 - 9e^{t^2}}$

7 $y = \pm\sqrt{Ax^2e^{x^2} - 1}$

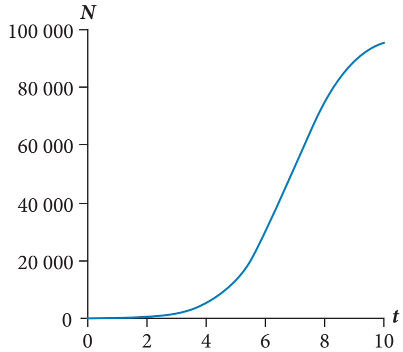
8 $y = 8.5e^{-2x} + 1.5$

9 $y = -\log_e [e^x(1 - x) + c]$

10 $y = -0.5 \log_e (c - e^x)$

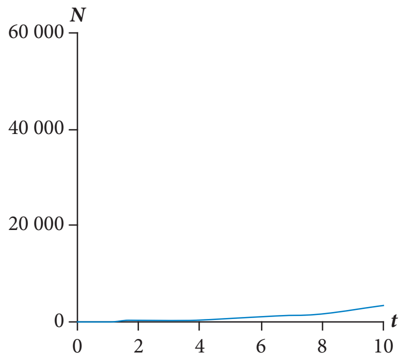
INVESTIGATION: EPIDEMICS

First model



With no vaccination, 95 000 are infected after 10 weeks. As the number vaccinated increases, the rate at which the disease spreads decreases.

Graph for 40% vaccination, so 60 000 are susceptible.



When 60% are vaccinated, only 490 are infected (out of the possible 40 000) after 10 weeks.
At 95% vaccination, only an extra 2 are infected after 10 weeks, even though 100 were infected at the start.
Vaccination reduces the speed with which a disease spreads through the population, so large-scale vaccination helps everyone, not just those vaccinated.

8.06

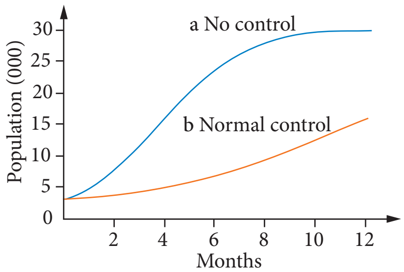
1 **a** 138 **b** 207 **c** 915

2 **a** About 6 years (5.973).
  b About 83 each subsequent year.

3 5.278 weeks (50 left), so they can be removed after 6 weeks.

4 22%

5 **a** 3000, 5000, 7941, 11 796, 16 152, 20 321, 23 722, 26 155, 27 734, 28 697, 29 262, 29 585, 29 768
  b 3000, 3500, 4070, 4718, 5446, 6259, 7159, 8143, 9208, 10 346, 11 547, 12 796, 14 077
  c



  d Some extra control measures will be needed.

6 **a** 22.7 million **b** 23.9 million
  c 26.3 million **d** 31.0 million

7 **a** **i** 4.6 g **ii** 5.5 g **iii** 7.4 g
  b 15.5%

8 28 904

9 50.45 min

10 9 weeks (8.47)

8.07

1 2.13 m/s^2 at $\tan^{-1}\left(\frac{7}{8}\right) \approx 41.2°$

2 1.8 m/s^2 at $\tan^{-1}\left(\frac{5}{2}\right) \approx 68.2°$

3 $20\sqrt{5}$ N at $\tan^{-1}(2) \approx 63.4°$

4 a 8.17 m/s^2 b 40.83 m/s c 102.1 m

5 a 200 N-s b 25 m/s

6 4125 N

7 a $\Delta\mathbf{p} = 300$ N-s to the east
   b About 18 m/s in the direction N 56° E

8 About 1.8 N (so the bird has a mass of about 180 g)

9 About 1 122 000 N in the direction N 11.2° W

10 a 0.285 kg-m/s north
   b 0.3705 kg-m/s at a bearing of 120°.
   c About 5.89 m/s at a bearing of about 253°.

8.08

1 -3 m/s^2, 37.5 m

2 a 3.75 m/s^2 b 14.6 m/s

3 50 s, 2500 m

4 36 m/s, 540 m

5 a 6 m/s b 2 m/s^2

6 a 132 m b 5 m/s

7 Proof

8 a 1 m/s^2 b 12.5 m

9 20 m/s

10 78.4 m

11 a About 0.19 m/s^2.
   b About 11.5 m/s (which is unrealistic).
   c About 22 m to the left.

12 a About 52 000 N perpendicular to the direction of travel towards the windward side.
   b About 7.1 m/s^2.

8.09

1 a $v = t^2 - \frac{5t^3}{3}$
   b $v = -\frac{1}{9}\left[\cos(3t-2) - \cos(2)\right]$
   c $v = \frac{15}{16}(e^{0.4t+5} - e^5) + 3$
   d $v = 7e^{\frac{3}{2}t}$

2 a $v = \sqrt{\sin(x) + 16}$ b $v = \sqrt{\frac{2}{5}x^2 - \frac{14}{5}x + 4}$
   c $v = \frac{1}{2}x + 3$ d $v = 4 - 2e^{-x}$
   e $v = \sqrt{\frac{2}{3}\log_e(x+1) + 4}$

3 $113\frac{1}{3}$ m

4 -93 m s^{-1}

5 11.48 m

6 a $k = 0.003\,456$ b 3.24 s
   c 30.25 m

7 a $v = -25 + 39e^{-\frac{t}{25}}$ b 72.1 m

8 a $v = ue^{-\frac{k}{m}t}$ b $x = \frac{m}{k}u\left(1 - e^{-\frac{k}{m}t}\right)$
   c Limiting distance $= \frac{mu}{k}$

9 $x = \frac{m}{2k}\log_e\left(1 + \frac{ku^2}{mg}\right)$

10 $h = \frac{mu}{k} - \frac{m^2 g}{k^2}\log_e\left(1 + \frac{ku}{mg}\right), t_h = \frac{m}{k}\log_e\left(1 + \frac{ku}{mg}\right)$

11 a Proof b $\frac{1}{2k}\log_e\left(\frac{4}{3}\right)$

8.10

1 $\frac{1}{4}$ s, 4 s^{-1}

2 0.67 s, 1.5 s^{-1}

3 a $\frac{1}{3} \text{ s}^{-1}$ b 1.57 m s^{-1} c 1.17 m s^{-1}
   d 3.29 m s^{-2} e 2.19 m s^{-2}

4 a 10 cm
   b 15.7 cm s^{-1}, 0 cm s^{-2}
   c 0 cm s^{-1}, 24.7 cm s^{-2}
   d 14.4 cm s^{-1}, 9.87 cm s^{-2}

5 a $\pi\sqrt{2}$ s b $\frac{\pi}{2}$ s

6 a $1.5\sqrt{2} \text{ m s}^{-1}$ b $0.9\sqrt{2} \text{ m s}^{-2}$

7 8 s, $28\sqrt{2}$ cm

8 $0.9\pi \text{ m s}^{-1}$, $0.9\pi^2 \text{ m s}^{-2}$

9 $\frac{3}{5\pi} \text{ m s}^{-1}$, $\frac{6}{25\pi^2}$ m

10 a 6.84 cm b 0.053 s

11 a Proof b 3 m, π s

12 0.0694 m, 8.33 m s^{-1}

13 a Proof b $a = 2, T = \pi$

14 12:46 p.m.

CHAPTER 8 REVIEW

1 D

2 E

3 B

4 A

5 A

6 E

7 B

8 C

9 $-\frac{3\sqrt{3}}{2}$, $3\sqrt{3}x + 2y - 24 = 0$

10 About 5.97 m/s.

11 $y = \frac{1}{2}\log_e\left|\frac{x-1}{x+1}\right| + 3$

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12 $y = 8e^{-0.5x}$

13 $P = \pm\sqrt{\dfrac{3}{c - 2V^3}}$

14 300 000 N

15 a About 0.82 s. b About 13.5 m/s.

16 The heavier car wins by 5.42 m.

17 $2\frac{1}{3}$ m/s

18 $v = \sqrt{\frac{1}{3}(2x^2 - 2x + 27)}$

19 a $x = 10\cos\left(\frac{\pi t}{6}\right)$ b 5.24 mm/s

c 12 s, 10 mm d 5.24 mm/s

e 2.74 mm/s^2

20 a 0.735 m b 1.28 s

21 12 m/s

22 About 10:52 a.m.

23 About 35 years

24 About 35.7 m

MIXED REVISION 3

Multiple choice

1 D
2 A
3 C
4 A
5 D
6 B

Short answer

1 $-\frac{1}{7}\log_e|7x-2|+c$

2 About 59.4 m (59.430…)

3 a About 0.5 m^2 (0.536…)

b 14 days (13.425…)

4 $-\frac{1}{3}\int u^7\,du$

Application

1 a $\int x^2 \sin(x)dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + c$

b $\int_0^{2\pi} x^2 \sin(x)dx = -4\pi^2$

2 a $\dfrac{x^2}{(x-1)^2(x+4)} = \dfrac{1}{5(x-1)^2} + \dfrac{9}{25(x-1)} + \dfrac{16}{25(x+4)}$

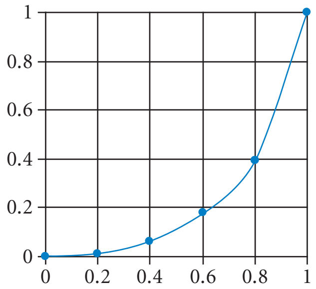
b $\frac{32}{25}\log_e(3) - \frac{57}{25}\log_e(2) + \frac{1}{5}$

3 11 days (10.674… days)

4 $12.5 - 12e^{-0.4t}$, 12.5 m/s

INVESTIGATION: LORENZ CURVES

x	0.2	0.4	0.6	0.8	1
Wealth	31203	191207	437856	766465	2215032
Cumulative	31203	222410	660266	1426731	3641763
Ratio	0.008568	0.061072	0.181304	0.391769	1



Income inequality coefficient for Australia ≈ 0.58
The coefficient has increased in recent years.
The coefficient is higher in most other countries.

9.01

1 0.5

2 $16\frac{1}{3}$

3 $57\frac{1}{6}$

4 36

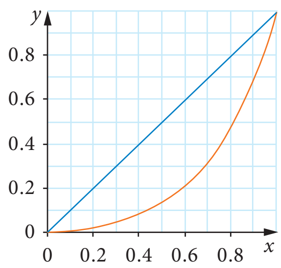
5 36

6 $127\frac{1}{54}$

7 $80\frac{1}{96}$

8 $49\frac{1}{3}$

9 a



b About 0.8%. c About 30.5%. d 0.515

10 $9\frac{23}{30}$

INVESTIGATION: POP-UP TENT

The volume is about 4 m^3.

9.02

1 a $\frac{243\pi}{20}$ b $\frac{25\pi}{7}$ c $2\pi[1-\log_e(2)]^2$

d $\frac{\pi e(e-1)}{4}$ e $\frac{\pi^2}{4}$

2 a 9π b $\frac{3\pi}{5\sqrt[3]{25}}$ c $\frac{\pi}{2}\left(e^2-1\right)$

d $\frac{\pi}{2}\left(4e-5\sqrt{e}\right)$ e approximately 7.17

f approximately 2.93

3 a D b $V=\frac{4}{3}\pi r^3$ where $r=5$. Vol $=\frac{500}{3}\pi$

4 E

5 C

6 B

7 $V=\frac{s^2h}{3}$

8 6000 m^3

9 $\frac{375\pi}{2}$

10 2021.84

9.03

1 C

2 D

3 A

4 E

5 C

6 E

7 a 4.8868 b 4.8756 c 4.8828

d 4.8830 e Simpson's rule is the closest.

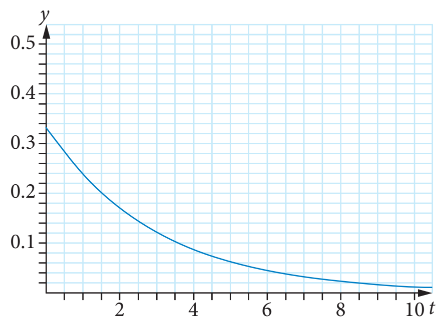
8 a 111.2969 b 111.4063 c $111\frac{1}{3}$ d $111\frac{1}{3}$

e Simpson's rule is exact.

9–11 Proofs

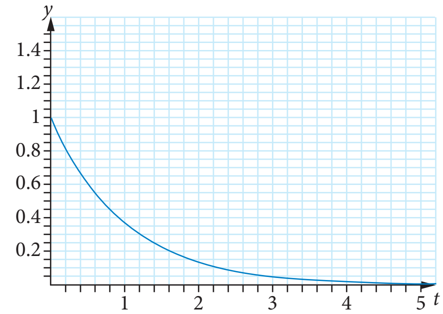
9.04

1 a



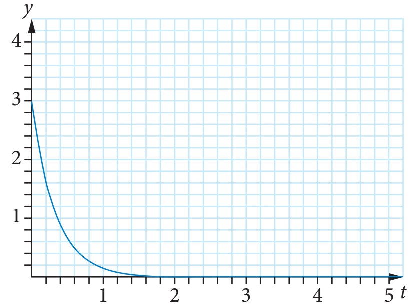
$\lambda=\frac{1}{3}$

b



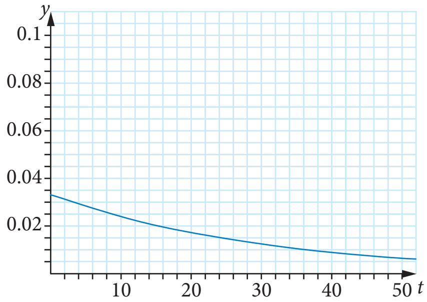
$\lambda=1$

c



$\lambda=3$

d



$\lambda=\frac{1}{30}$

The function with the higher value of λ starts higher and is skewed more to the left.

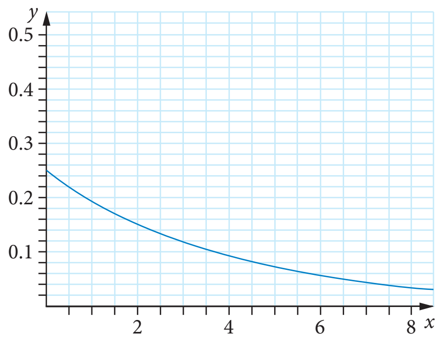
2 A

3 C

4 B

5 E

6 a



$\lambda=0.25$

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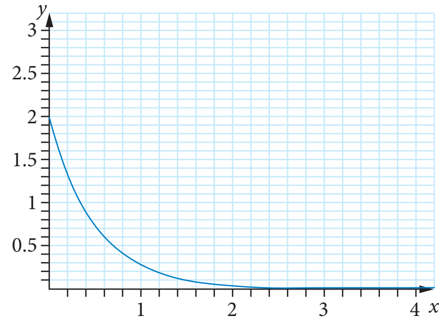
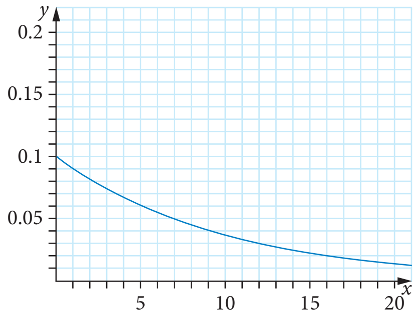
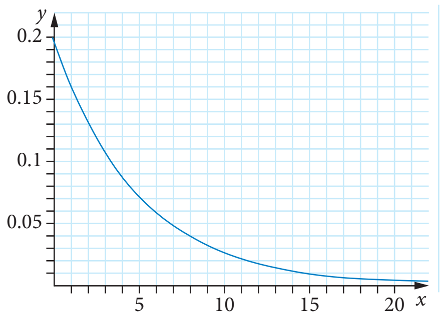


$\lambda = 2$

b

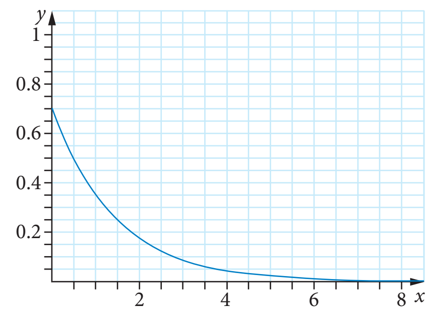


$\lambda = 0.1$

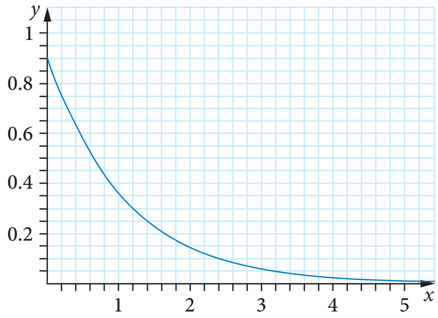


$\lambda = 0.2$

c



$\lambda = 0.7$



$\lambda = 0.9$

The function with the higher value of λ starts higher and is skewed more to the left.

7 **a** $1-e^{-\frac{9}{5}}$ **b** $e^{-\frac{9}{5}}-e^{-6}$ **c** $e^{-\frac{57}{10}}-e^{-\frac{63}{10}}$ **d** e^{-6}

8 **a** $1-e^{-\frac{9}{10}}$ **b** $e^{-\frac{6}{5}}-e^{-\frac{9}{5}}$ **c** $e^{-\frac{9}{10}}$ **d** $e^{-\frac{3}{2}}$

9 $\frac{2}{3}$

10 0.0625

11 **a** $P(T \geq 100) = e^{-\frac{1}{2}}$ **b** 200 hours

12 **a** $\lambda = \frac{2}{3}$ **b** 0.632 **c** 0.069

13 **a** About 63.2%. **b** About 1.73 years.
c About 20.2%.

9.05

1 **a** A **b** D

2 **a** **A** **b** D

3 B

4 C

5 20 minutes

6 **a** 2.4 s

b $\sigma = \frac{1}{25}$

c $\text{median} = \frac{\log_e(2)}{\lambda}$, so

$\text{median} = \frac{1}{25}\log_e(2) \approx 0.027\ 73$

d $\frac{1}{25}\log_e\left(\frac{4}{3}\right) \approx 0.01$ min

e $\frac{1}{25}\log_e(4) \approx 0.06$ min

7 **a** **i** $P(0 \leq x \leq 8) \approx 0.4353$ **ii** $P(8 \leq x) \approx 0.5647$

b Total = 1 as expected.

8 $\lambda = \sqrt{\log_e(5)}$

9 $P(0 \leq x \leq 5) = 0.9968$

10 $\frac{1}{4}\log_e \frac{100}{99}$

a $R(3) = 0.9925$ **b** $R(5) = 0.9875$

c $R(10) = 0.9752$

11 About 6.66 months

12 a $\frac{1}{3000}$ b 0.1889

13 a About 0.2835 b About 4.159

c About 0.811

CHAPTER 9 REVIEW

1 B

2 E

3 A

4 B

5 E

6 $\frac{9}{8}$ square units

7 121.5 square units

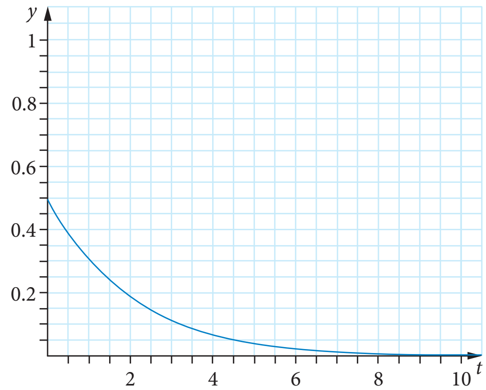
8 $78\frac{1}{12}$ square units

9 a $\frac{243\pi}{5}$ b $\pi e^4(e^{12} - 1)$

10 a $\frac{9\pi}{2}$ b $\frac{\pi}{8}\left(e^2 - 1\right)$

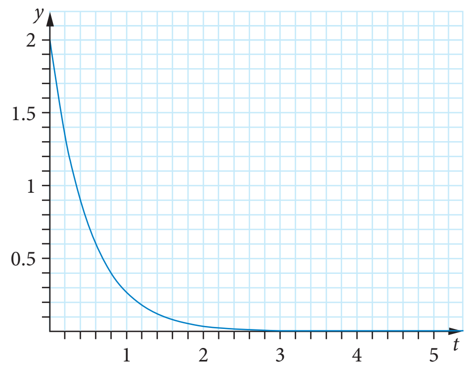
11 818.05

12 a $f(t) = \frac{1}{2}e^{-\frac{1}{2}t}$



$\lambda = \frac{1}{2}$

b $f(t) = 2e^{-2t}$



$\lambda = 2$

The lower the value of λ the flatter the slope is at the beginning.

13 a 15 b 30

14 $E(X) = 16 = \frac{1}{\lambda}$ so $\lambda = \frac{1}{16}$

15 a About 0.1353

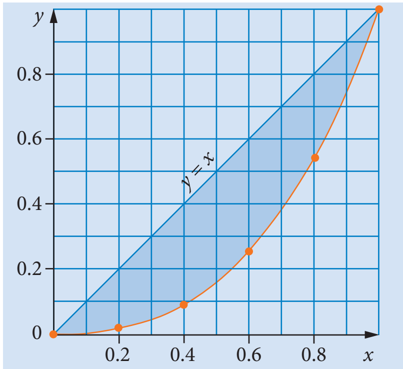
b About 92 (8.2 will last)

16 $f(t) = 0.08e^{-0.08t}$ with $\lambda \approx 0.08$

a i $P(X < 9) = 0.5132$ ii $P(X > 9) = 0.4868$

b Total of probabilities equal 1 as expected.

17 a



b 1.95% c About 0.453.

18 $52\frac{4}{15}$

19 $\sqrt{\log_e(10)}$

20 0.2231

INVESTIGATION: SAMPLE HEIGHTS

The standard deviation of the sample means is less than the standard deviation for the whole class.

The number of samples possible from your class is $2^n - 1$, where n is the number of people.

10.01

1 a 76.6 b 80.6 c 81.1 d 75.5

2 a 24.3 b 27.8 c 24.7 d 37.6

3 a Results will vary, but it will be close to 24.

b It will vary around 0.8.

c The sample mean is close to the mean, but the sample standard deviation is smaller.

4 a Results will vary, but it will be close to \$120.

b It will vary around \$6.

c The sample mean is close to the mean, but the sample standard deviation is smaller.

5 a Results will vary, but it will be close to 23 cm.

b It will vary around 0.5.

c The sample mean is close to the mean, but the sample standard deviation is smaller.

6 a Results will vary but it will be close to \$290.

b It will vary around \$13.

c The sample mean is close to the mean, but the sample standard deviation is smaller.

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7 a 7.84, 1.78
b Results will vary, mostly between 6.6 and 9.1.
c The mean will be close to 7.8 and the standard deviation will be close to 0.6.
d It will be close to 3.
e It is almost the same.

8 a 15.8, 5.4
b Results will vary, mostly between 12.8 and 18.8.
c The mean will be close to 15.8 and the standard deviation will be close to 1.4.
d It will be close to 4.
e It is almost the same.

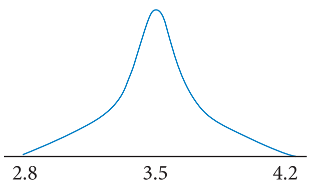
9 a 55.68, 19.65
b Results will vary, mostly between 48 and 64.
c The mean will be close to 56 and the standard deviation will be close to 4.
d It will be close to 5.
e It is almost the same.

10 a 0.38, 0.08
b Results will vary, mostly between 0.33 and 0.43.
c The mean will be close to 0.38 and the standard deviation will be close to 0.023.
d It will be close to 3.5.
e It is almost the same.

10.02

Specific answers for graphs in this exercise will vary, but patterns will be similar. The graph outlines show only the *general shapes* of the histograms and dot graphs.

1 a, b Similar to:

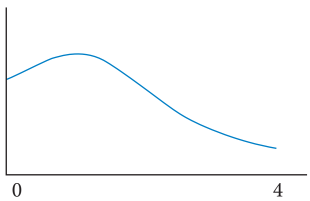


c The graph is not a uniform distribution like the original. It is more bell-shaped and extends only from about 2.8 to 4.2.

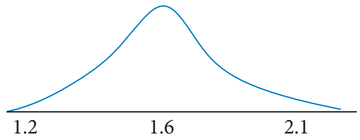
2 a

	2	3	4	5	6
2	0	1	2	3	4
3	1	0	1	2	3
4	2	1	0	1	2
5	3	2	1	0	1
6	4	3	2	1	0

b Similar to:

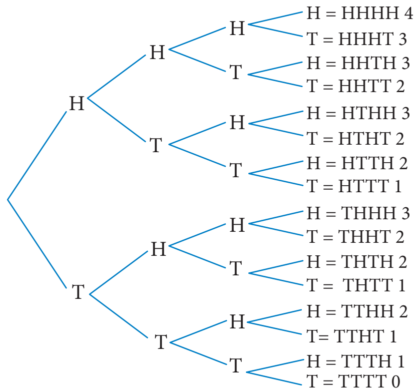


c, d Similar to:

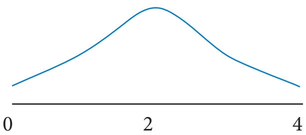


e The graph of a single sample is unevenly spread from 0 to 4 with a peak at 0 or 1 and is skewed to the right.
The sampling distribution is spread from about 1.1 to 2.2 with a peak at about 1.6. It is bell-shaped.

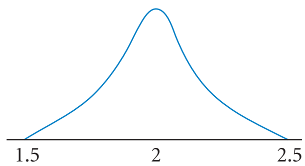
3 a



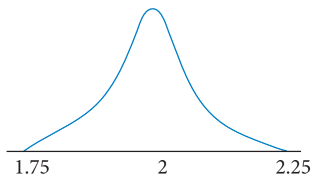
b Similar to:



c, d Similar to:

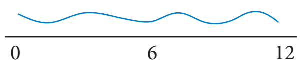


e, f Similar to:



g The single sample graph has a central peak, but the sampling distributions are more bell-shaped and much more concentrated.

4 a Uneven.



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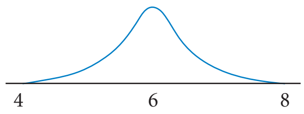
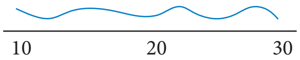
b, c Similar to:

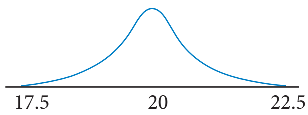


d The single sample is distributed from 0 to 12, but the sampling distribution is only from 4 to 8 with a peak about 6 in a bell-shaped curve.

5 a Uneven.

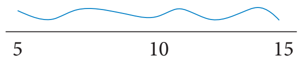


b, c Similar to:

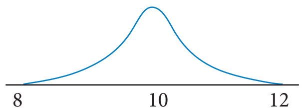


d The single sample is distributed from 10 to 30, but the sampling distribution is only from 17.5 to 22.5 with a peak about 20 in a bell-shaped curve.

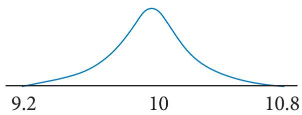
6 a Uneven.



b, c Similar to:

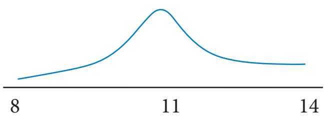


d, e Similar to:

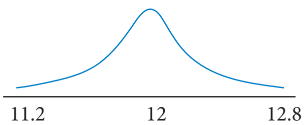


f The single sample is distributed from 5 to 15, but the sampling distributions are only from 8 to 12 and 9.2 to 10.8, with peaks at about 10 in bell-shaped curves. The larger sample size has a more concentrated distribution.

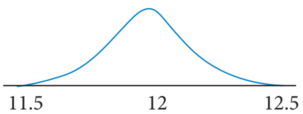
7 a Similar to:



b, c Similar to:

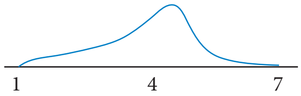


d, e Similar to:

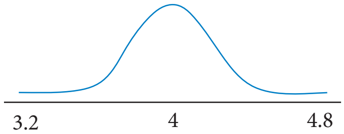


f The single sample is unevenly distributed from about 8 to 14, but the sampling distributions are only from 11.2 to 12.8 and 11.5 to 12.5, with peaks at about 12 in bell-shaped curves. The larger sample size has a more concentrated distribution.

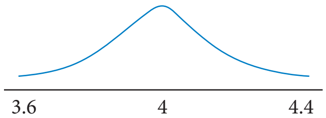
8 a Uneven.



b, c Similar to:



d, e Similar to:



f The single sample is unevenly distributed from about 1 to 7, but the sampling distributions are only from 3.2 to 4.8 and 3.6 to 4.4, with peaks at about 4 in bell-shaped curves. The larger sample size has a more concentrated distribution.

10.03

1 About 25, 0.6
2 About 80, 0.8
3 About $200, 1500
4 About 13.4
5 About 360
6 About 255 and 38.7
7 0.009
8 0.0368
9 0.7688
10 About $500 000

10.04

1 [−1.96, 1.96]
2 [−2.57, 2.57]
3 [−1.28, 1.28]
4 [75.3, 124.7]
5 [46.5, 93.5]
6 [8.6, 101.4]
7 About 84.7%.
8 About 95.4%.
9 About 78.9%.
10 8 months (8.545…)

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INVESTIGATION: CONFIDENCE INTERVALS AND THE MEAN

The expected standard deviation of the sampling distribution is about 1.15.
The result for the class is likely to be between about 8% and 12%.

10.05

1 [25.5, 30.5]
2 [129, 141]
3 [63.5, 70.5]
4 [415.7, 424.3]
5 [–0.02, 1.42]
6 [49.1, 54.9]
7 a 23.95, 11.81
b About [20.9 ha, 27 ha].
c Only Victorian potato farms have been used.
8 a 743, 85.7
b About [715, 771].
c The data is not a random sample.
9 a \$1557.20, \$150.83
b About [\$1508, \$1607].
c The sample is less than 30.
10 a 10.4, 11.7
b About [6.8, 14.1].
c There are a lot of 'round figures' in the data that suggest some students may have been giving the recommended times at their schools instead of the actual times.

INVESTIGATION: SURVEY IN SCHOOL

Results will vary.

10.06

1 a [7.9, 9.1], [7.2, 8.4], [6.9, 8.1]
b Even at the 90% confidence level, the top 3 overlap so the difference could just be sampling variation.
2 a [14.9 years, 15.1 years], [23.8 months, 24.2 months]
b The samples are far from random and are likely to be very unrepresentative because of self-selection bias.
3 a 13 months
b 13.8 months
c The mean would be 9.1 months and the standard deviation would be 13 months.
d If you count only the people who finished, it is justified, but maybe the ones who dropped out did so because they weren't benefitting..
4 a 7.7 mm b 0.23
c No, because if it is random, it could be very high in any year. However, we do know that rainfall is not entirely random (e.g., El Nino).
5 a 0.78°C b 7%
c It is assumed that the data over the last 12 years is a random sample, and the sample size is too small to approximate using the normal distribution.

CHAPTER 10 REVIEW

1 E
2 C
3 A
4 B
5 E
6 C
7 A
8 The graph is likely to be uneven from 1 to 8, but with no particular peak.
9 The graph will probably be a bell-shaped curve with the centre at 6.5 but not extending over the whole domain (probably about 5.7 to 7.3).
10 The centres of all three graphs will be at about 5. **I** will be skewed to the right, while **II** and **III** will be more symmetrical. The spread of the results will decrease from **I** to **II** to **III**.
11 $\mu = 14, \sigma_{\bar{x}} \approx 0.26$
12 About 0.275.
13 [–1.645, 1.645]
14 [39.4, 80.6]
15 About 0.959.
16 [37.7, 42.3]
17 [\$1.66, \$3.34]
18 a [\$124.15, \$145.85]
b The sample is not random because only students from one school were used.
19 a [\$118.51, \$121.49]
b The poll is likely to be biased towards people who want fast access because mobile phones are used more by younger people than older people. It is not random.
20 a $\mu = 8, \sigma \approx 1.6$
b The results mean that only 7 or 8 out of the 247 surveyed rated the security less than 5 out of 10, so probably not a cause for concern.

MIXED REVISION 4

Multiple choice

1 D
2 C
3 B
4 A
5 A
6 D

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Short answer

1 $\mu = 10$, $\sigma_{\bar{x}} = \frac{\sqrt{30}}{6} \approx 0.9129$

2 $\text{Area} = \int_{-4}^{1} 4 - \left(x^2 + 3x\right) dx$

$= \int_{-4}^{1} 4 - x^2 - 3x dx$

$= \left[4x - \frac{1}{3}x^3 - \frac{3}{2}x^2\right]_{-4}^{1}$

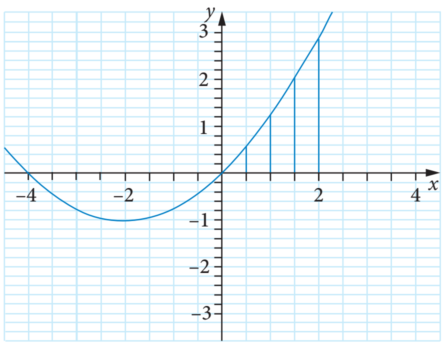
$= \left(4 - \frac{1}{3} - \frac{3}{2}\right) - \left(-16 + \frac{64}{3} - 24\right)$

$= \frac{125}{6}$

$\text{Area} = 20\frac{5}{6}$ square units

3 [22.7, 26.7]

4 $f(x) = \frac{1}{4}x^2 + x$ from $x = 0$ to $x = 2$ with strips $\frac{1}{2}$ unit wide

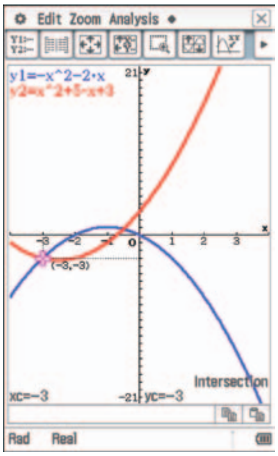


$\text{Area} = \frac{1}{2}\left[f(0.25) + f(0.75) + f(1.25) + f(1.75)\right]$

$\text{Area} = \frac{85}{32}$ square units

Application

1 a



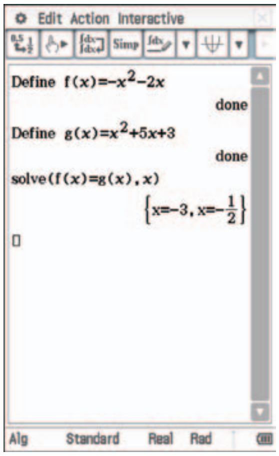
$f(x) = -x^2 - 2x$ and $g(x) = x^2 + 5x + 3$.

b Solve $-x^2 - 2x = x^2 + 5x + 3$

gives $-2x^2 - 7x - 3 = 0$

so $2x^2 + 7x + 3 = 0$ and $(2x + 1)(x + 3) = 0$

so $x = -3$, $x = -\frac{1}{2}$



c $\text{Area} = \int_{-3}^{-\frac{1}{2}} \left(-x^2 - 2x\right) - \left(x + 5x + 3\right) dx$

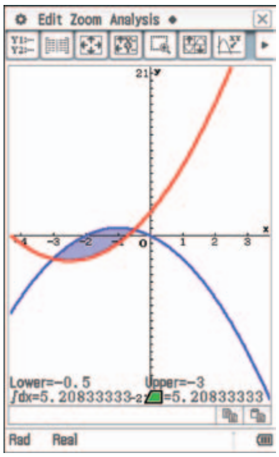
$= \int_{-3}^{-\frac{1}{2}} -2x^2 - 7x - 3\, dx$

$= \left[-\frac{2}{3}x^3 - \frac{7}{2}x^2 - 3x\right]_{-3}^{-\frac{1}{2}}$

$= \left(\frac{1}{12} - \frac{7}{8} + \frac{3}{2}\right) - \left(18 - \frac{63}{2} + 9\right)$

$= \frac{125}{24}$

$\text{Area} = \frac{125}{24}$ square units



2 a [17.4, 25.3]

b The sample is not random and there are less than 30 people in the sample.

3 The probability of failure is less than 7% for about 69 h.

4 0.1769

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